

Undergraduate Texts in Mathematics

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Linear Algebra



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***Dedicated to Mi-Soo
without whom this book and
many other things could not exist***

Preface

This text is written for a course in linear algebra at the (U.S.) sophomore undergraduate level, preferably directly following a one-variable calculus course, so that linear algebra can be used in a course on multidimensional calculus. Realizing that students at this level have had little contact with complex numbers or abstract mathematics the book deals almost exclusively with real finite-dimensional vector spaces in a setting and formulation that permits easy generalization to abstract vector spaces. The parallel complex theory is developed in the exercises.

The book has as a goal the principal axis theorem for real symmetric transformations, and a more or less direct path is followed. As a consequence there are many subjects that are not developed, and this is intentional.

However a wide selection of examples of vector spaces and linear transformations is developed, in the hope that they will serve as a testing ground for the theory. The book is meant as an *introduction* to linear algebra and the theory developed contains the essentials for this goal. Students with a need to learn more linear algebra can do so in a course in abstract algebra, which is the appropriate setting. Through this book they will be taken on an excursion to the algebraic/analytic zoo, and introduced to some of the animals for the first time. Further excursions can teach them more about the curious habits of some of these remarkable creatures.

Göttingen,
December 1977

LARRY SMITH

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