Undergraduate Texts in Mathematics

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A First Course in Real Analysis

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Preface

The first course in analysis which follows elementary calculus is a critical one for students who are seriously interested in mathematics. Traditional advanced calculus was precisely what its name indicates—a course with topics in calculus emphasizing problem solving rather than theory. As a result students were often given a misleading impression of what mathematics is all about; on the other hand the current approach, with its emphasis on theory, gives the student insight in the fundamentals of analysis.

In *A First Course in Real Analysis* we present a theoretical basis of analysis which is suitable for students who have just completed a course in elementary calculus. Since the sixteen chapters contain more than enough analysis for a one year course, the instructor teaching a one or two quarter or a one semester junior level course should easily find those topics which he or she thinks students should have.

The first Chapter, on the real number system, serves two purposes. Because most students entering this course have had no experience in devising proofs of theorems, it provides an opportunity to develop facility in theorem proving. Although the elementary processes of numbers are familiar to most students, greater understanding of these processes is acquired by those who work the problems in Chapter 1. As a second purpose, we provide, for those instructors who wish to give a comprehensive course in analysis, a fairly complete treatment of the real number system including a section on mathematical induction.

Although Chapter 1 is useful as an introduction to analysis, the instructor of a short course may choose to begin with the second Chapter. Chapters 2 through 5 cover the basic theory of elementary calculus. Here we prove many of the theorems which are "stated without proof" in the standard freshman calculus course.

Crucial to the development of an understanding of analysis is the concept of a metric space. We discuss the fundamental properties of metric spaces in Chapter 6. Here we show that the notion of compactness is central and we prove several important results (including the Heine–Borel theorem) which are useful later on. The power of the general theory of metric spaces is aptly illustrated in Chapter 13, where we give the theory of contraction mappings and an application to differential equations. The study of metric spaces is resumed in Chapter 15, where the properties of functions on metric spaces are established. The student will also find useful in later courses results such as the Tietze extension theorem and the Stone-Weierstrass theorem, which are proved in detail.

Chapters 7, 8, and 12 continue the theory of differentiation and integration begun in Chapters 4 and 5. In Chapters 7 and 8, the theory of differentiation and integration in $\mathbb{R}_N$ is developed. Since the primary results for $\mathbb{R}_1$ are given in Chapters 4 and 5 only modest changes were necessary to prove the corresponding theorems in $\mathbb{R}_N$. In Chapter 12 we define the Riemann–Stieltjes integral and develop its principal properties.

Infinite sequences and series are the topics of Chapters 9 and 10. Besides subjects such as uniform convergence and power series, we provide in Section 9.5 a unified treatment of absolute convergence of multiple series. Here, in a discussion of unordered sums, we show that a separate treatment of the various kinds of summation of multiple series is entirely unnecessary. Chapter 10 on Fourier series contains a proof of the Dini test for convergence and the customary theorems on term-by-term differentiation and integration of such series.

In Chapter 14 we prove the Implicit Function theorem, first for a single equation and then for a system. In addition we give a detailed proof of the Lagrange multiplier rule, which is frequently stated but rarely proved. For completeness we give the details of the proof of the theorem on the change of variables in a multiple integral. Since the argument here is rather intricate, the instructor may wish to assign this section as optional reading for the best students.

Proofs of Green's and Stokes' theorems and the divergence theorem in $\mathbb{R}_2$ and $\mathbb{R}_3$ are given in Chapter 16. The methods used here are easily extended to the corresponding results in $\mathbb{R}_N$.

This book is also useful in freshman honors courses. It has been our experience that honors courses in freshman calculus frequently falter because it is not clear whether the honors student should work hard problems while he learns the regular calculus topics or should omit the regular topics entirely and concentrate on the underlying theory. In the first alternative, the honors student is hardly better off than the regular student taking the ordinary calculus course, while in the second the honors student fails to learn the *simple* problem solving techniques which, in fact, are useful later on. We believe that this dilemma can be resolved by employing two texts—one a standard calculus text and the other a book such as this one which provides the theoretical basis of the calculus in one and several dimensions. In this way the honors student gets both theory and practice. Chapters 2 through 5 and Chapters 7 and 8 provide a thorough account of the theory of elementary calculus which, along with a standard calculus book, is suitable as text material for a first year honors program.

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