

Applied Mathematical Sciences | Volume 1

F. John

Partial Differential Equations

Second Edition

With 31 Illustrations



Springer-Verlag New York · Heidelberg · Berlin
1975

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AMS Classifications: 35-02, 35A10, 35EXX, 35L05, 35L10

Library of Congress Cataloging in Publication Data

John, Fritz, 1910-

Partial differential equations.

(Applied mathematical sciences, v. 1)

Bibliography: p.

Includes index.

1. Differential equations, Partial. I. Series.

QA1.A647 vol. 1, 1975 [QA374] 515'.353 74-26827

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ISBN-13: 978-0-387-90111-4
DOI: 10.1007/978-1-4615-9979-1

e-ISBN-13: 978-1-4615-9979-1

PREFACE

These Notes grew out of a course given by the author in 1952-53. Though the field of Partial Differential Equations has changed considerably since those days, particularly under the impact of methods taken from Functional Analysis, the author feels that the introductory material offered here still is basic for an understanding of the subject. It supplies the necessary intuitive foundation which motivates and anticipates abstract formulations of the questions and relates them to the description of natural phenomena.

Added to this second corrected edition is a collection of problems and solutions, which illustrate and supplement the theories developed in the text.

Fritz John
New York
September, 1974

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