

**MULTICRITERION
DECISION
IN
MANAGEMENT**

*Principles and
Practice*

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MULTICRITERION DECISION IN MANAGEMENT

*Principles and
Practice*

by

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FOREWORD

Why another book on multicriterion decision making?

Several answers can be given to this question. The first is that, as far as we know, there exists at present no book devoted exclusively to discrete multicriterion decision making. Within companies and organizations, multicriterion analysis is used in two different ways: there is discrete multicriterion decision making, which is concerned with choice among a finite number of possible alternatives such as projects, investments, decisions etc., and it is this domain which is the subject of the book. Then there is the other main area, a subject which we shall not be treating here: multiple criteria linear programming. Here, the approach is to extend the results of linear programming and the associated algorithms (the most well known of which are simplex, gradient and Karmarkar) to multiple criteria.

The first answer to our question, then, is that a book on one of the two components of multicriterion analysis which can be of practical help in company decision making and management should be welcome, and this book is foremost written for those who want to apply the methods of multicriterion analysis. *Thus, starting from the main scientific results which are the foundation of the domain, we will show the principle methods of multicriterion analysis, their advantages and their shortcomings.* Although the presentation is rigorous, it should be accessible to all readers who have given some thought to decision problems; it is designed to be put into practice by managers and decision makers in the course of their daily professional life.

The second answer to the question is that, despite the large body of scientific papers on multicriterion decision making, the tools, methods and thinking behind multicriterion analysis remain virtually unknown to managers and engineers at all levels. There are several reasons for this. The main one would appear to be cultural; at the present time – in Europe, at least, where this book was written – the scientific culture still favors the notion of the 'best decision', whereas, as we shall see, in multicriterion analysis any optimum in the strict sense of the term does not exist. Herein lies the original sin which is at the root of the almost total absence of the discipline in academic curricula and therefore the understandable ignorance of the subject by engineers and managers. It is true that multicriterion analysis lacks the huge body of mathematical results which make optimization so attractive, but it can nevertheless offer several very interesting properties, ignorance of which leads decision makers to re-invent the wheel every day. More seriously, in companies this ignorance results in an impoverishment of thinking, a kind of self-censoring where

all too often, analysis halts at the first convenient optimization which is quite clearly a pale shadow of the real complexity of the situation.

We believe that that is the main reason for using multicriterion analysis; it is no longer possible to ignore the fact that each real decision is the result of a compromise between several solutions which all have their advantages and disadvantages, depending on one's point of view. In organizations in future, it will become harder and harder to disregard the complexity of points of view, motivations and objectives. The day of the single objective (profit, social, environment etc.) is over, and the wishes of all those involved in all their diversity must be taken into account; and to do this, a minimum knowledge of multicriterion analysis is necessary. One of the objectives of this book is to supply that knowledge and enable it to be applied.

Acknowledgements

This book incorporates material from lectures and research work carried out by the authors in the last twenty years in their respective universities.

At the Université P. et M. Curie in Paris, the lectures were partly to fourth year undergraduates, partly to post-graduates in the specialty 'Decision methods and algorithms' under J.-Y. Jaffray. Large amounts of material have been borrowed from this teaching, which began in 1975 and continues today. Chapters 2, 3 and 6 in particular re-state the demonstrations and ideas coming directly from J.-Y. Jaffray's lectures on decision theory; we would like to express our gratitude to him for his permission to use the material and for sharing with us his extremely erudite knowledge of the subject.

In 1987, inside the venerable walls of the Universidad de Alcalá de Hénare, the authors held a summer school on multicriterion decision making. This agreeable experience was, through the interest shown by the participants and the support of the University authorities, at the origin of this book. Every year a doctoral course on multicriterion decision making is given at Alcalá by S. Barba-Romero, who would like to express his appreciation for the financial support from the Spanish National Research Plan (CAICYT project 499/84) given to the multicriterion decision making research team which he directs in the University of Alcalá. Among the members of this team, we would especially like to thank J. Pérez Navarro, who checked several chapters of the book.

In the field of multicriterion analysis research, the work of B. Roy is paramount. The LAMSADE laboratory (Université de Paris-Dauphine), of which he is Director, is the leading research center in the domain in France; we cannot stress too greatly how invaluable his help and support have been. The quality of the LAMSADE documentation center and the intensity of research activities taking place there make a visit obligatory for all who are interested in multicriterion analysis. We are extremely grateful to B. Roy for the warm welcome we have always received at LAMSADE, and we would particularly like to thank D. Champ-Brunet for his cooperation in the bibliographic search which was vital to the achievement of this book. Finally, we thank M.-J. Pomerol for her help in the final stages.

THE MAIN SYMBOLS USED IN THIS WORK

A	choice set	p. 18
P or \succ	strictly preferred to	p. 21
I or \approx	indifferent to	p. 21
Q or \succsim	preferred or indifferent to	p. 21
NC	not comparable to	p. 21
\mathcal{H}	related pairs	p. 21
$\overline{\mathcal{R}}$	non-related pairs	p. 21
R	binary relation	p. 21
$ \mathbb{R}$	set of real numbers	p. 27
P_f or \succ_f	weak preference	p. 46
C	complete set	p. 60
O	set of efficient points	p. 61
A^c	convex envelope of A	p. 84
\mathcal{PR}	set of preorders	p. 124
x_{-i}	excluding coordinate i	p. 156
ϕ	flow in a graph	p. 197