

# APPLIED STRESS ANALYSIS OF PLASTICS

A Mechanical Engineering Approach

# APPLIED STRESS ANALYSIS OF PLASTICS

A Mechanical Engineering Approach

S.I. Krishnamachari

L.J. Broutman & Associates, Ltd.  
Chicago, IL



SPRINGER SCIENCE+BUSINESS MEDIA, LLC

*To my mother.*

Copyright © 1993 by Springer Science+Business Media New York  
Originally published by Van Nostrand Reinhold in 1993  
Softcover reprint of the hardcover 1st edition 1993  
Library of Congress Catalog Card Number 92-27825  
ISBN 978-0-442-23907-7

All rights reserved. No part of this work covered by the copyright hereon may be reproduced or used in any form by any means—graphic, electronic, or mechanical, including photocopying, recording, taping, or information storage and retrieval systems—without written permission of the publisher.

16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

**Library of Congress Cataloging-in-Publication Data**

Krishnamachari, S. I. (Sadagopa I.), 1944-

Applied stress analysis of plastics: a mechanical engineering approach /  
S. I. Krishnamachari

p. cm.

Includes bibliographical references and index..

ISBN 978-0-442-23907-7 ISBN 978-1-4615-3110-4 (eBook)

DOI 10.1007/978-1-4615-3110-4

1. Plastics—Testing. 2. Stresses and strains. I. Title.

TA455.P5K7 1992

620.1'9233'0287—dc20

92-27825

CIP

# Contents

PREFACE	<i>xi</i>
ACKNOWLEDGMENTS	<i>xiv</i>
<b>1. BASIC CONCEPTS: PERSPECTIVES IN ELASTICITY THEORY</b>	<b><i>1</i></b>
1.1 Introduction	<i>1</i>
1.2 Uniqueness of Plastics Stress Analysis	<i>2</i>
1.3 Similarities between Plastics and Metals	<i>4</i>
1.4 Significance of Calculated Stress and Strain	<i>4</i>
1.5 The Basic Complexity of Stress Analysis	<i>7</i>
1.6 What Is Stress?	<i>7</i>
1.7 What Is Strain?	<i>11</i>
1.8 Commentary on the Definitions	<i>16</i>
1.9 Equilibrium	<i>17</i>
1.10 Hooke's Law	<i>19</i>
1.11 Plane Stress and Plane Strain	<i>21</i>
1.12 Analysis of Stress at a Point	<i>22</i>
1.13 Representation by Matrices	<i>27</i>
1.14 Transformation Using Matrices	<i>28</i>
1.15 Compatibility	<i>44</i>
1.16 Framework of Linear Elasticity Theory	<i>44</i>
References	<i>55</i>
Exercises	<i>55</i>

<b>2. APPLICATIONS OF LINEAR ELASTIC BEHAVIOR</b>	<b>63</b>
2.1 Introduction	63
2.2 Bending of Beams	64
2.3 The Unit Load Method (ULM)	80
2.4 Application to Piping Flexibility Analysis	88
2.5 Problems in Polar Coordinates	95
2.6 Thick Pressurized Pipe	98
2.7 Rotating Cylinders	111
2.8 Axisymmetric Shell Problems	116
2.9 Structural Discontinuity—The Concept	127
2.10 Applications in the Theory of Plates	132
References	136
Exercises	138
<b>3. BEYOND ELASTIC BEHAVIOR</b>	<b>146</b>
3.1 Introduction	146
3.2 Onset of Yield	149
3.3 Post-Yield Stress-Strain Relationship	163
3.4 Crazeing	164
References	166
Exercises	166
<b>4. RATIONALE OF STRESS ANALYSIS</b>	<b>168</b>
4.1 Design by Analysis	168
4.2 Objectives of Stress Analysis	169
4.3 Factor of Safety (FOS)	174
4.4 Basis for Factor of Safety	183
4.5 Integration of Stress Analysis with Design	184
4.6 Stress Categories	186
4.7 How to Identify Stress Categories	191
References	196
Exercises	197
<b>5. APPLIED VISCOELASTICITY</b>	<b>200</b>
5.1 Introduction	201
5.2 Aspects of Viscoelasticity	201
5.3 Viscoelastic Models	206
5.4 Spring Dashpot Models	207
5.5 The Time Spectra Concept	221
5.6 Dynamic Behavior of Linear Viscoelastic Materials	222
5.7 Boltzmann's Superposition Principle	231

5.8	Use of Laplace Transforms in BSP	240
5.9	The Correspondence Principle	242
5.10	Correspondence Principle for 3-D Viscoelasticity	245
5.11	Pseudoelasticity	251
5.12	An Interim Study	254
5.13	Comments on the Use of Pseudoelasticity	255
5.14	Findley's Constants	259
5.15	Methods of Determining $E(T)$	269
5.16	Concluding Remarks	272
	References	273
	Exercises	274
<b>6.</b>	<b>FRACTURE MECHANICS</b>	<b>277</b>
6.1	Introduction	277
6.2	An Outline	278
6.3	Strain Energy Release Rate Criterion	279
6.4	Stress Analysis of Cracks	282
6.5	The $K_{Ic}$ or the Stress Intensity Criterion	285
6.6	The $J$ -Integral Criterion	285
6.7	The CTOD Criterion	287
6.8	Remarks on the Fracture Criteria	287
6.9	More about $G$	289
6.10	Calculation of $K$	295
6.11	A Few Useful Results	296
6.12	Principle of Superposition for Calculating $K$	298
6.13	Concept of Leak-Before-Break	313
6.14	Fracture Toughness for Light Weight Designs	316
6.15	Effects of Crack Tip Plasticity	317
6.16	Shape of Plastic Zone	320
6.17	Accounting for Plastic Effects	322
6.18	Contained Plasticity	323
6.19	Crack Opening Displacement (COD)	326
6.20	Fracture Initiation Process—Crazing	328
6.21	Fatigue	331
6.22	Conclusion	336
	References	336
	Exercises	337
<b>7.</b>	<b>REINFORCED PLASTICS</b>	<b>340</b>
7.1	Motivation	342
7.2	Hooke's Law for Orthotropy	345

- 7.3 Micromechanics—Moduli of Composites 354
- 7.4 Micromechanics—A Summary 369
- 7.5 Macromechanics of a U.D.L. 371
- 7.6 Transformation of Elastic Moduli 371
- 7.7 Calculation of Stresses in a 1-2 System 383
- 7.8 The Meaning of  $\nu$ 's and  $\eta$ 's 383
- 7.9 Failure Criteria 389
- 7.10 Factor of Safety (FOS) 398
- 7.11 Failure Envelopes 402
- 7.12 Mechanics of Laminated Plates 407
- 7.13 Stress Analysis of a Laminate Point 408
- 7.14 Symmetric Laminates 427
- 7.15 Quasi-Isotropic Laminates 430
- 7.16 Hygrothermal (HT) Effects 437
- 7.17 Hygro-Thermal Stresses 439
- 7.18 Conclusion 442
- References 444
- Exercises 445

**8. FINITE ELEMENT METHOD: AN INTRODUCTION 449**

- 8.1 Motivation 449
- 8.2 Overview of FEM 450
- 8.3 Basics of FE Stress Analysis 454
- 8.4 Discretization 459
- 8.5 Interpolation of Displacements 463
- 8.6 Calculation of Element Stiffness 472
- 8.7 Calculation of Element Load Vectors 479
- 8.8 Assembly of the Global Stiffness Matrix 482
- 8.9 The Nature of the Global Stiffness Matrix—[K] 485
- 8.10 Displacement Boundary Conditions 489
- 8.11 Solution of the Unknown Displacements 490
- 8.12 Reactions, Strains, and Stresses 499
- 8.13 Post-Processing 501
- 8.14 Isoparametric Elements 503
- 8.15 The Gauss Quadrature 510
- 8.16 Conclusion 514
- References 514
- Exercises 515

**9. GUIDELINES FOR FE ANALYSIS 522**

- 9.1 Capabilities of a Modeling Software 522
- 9.2 Do's and Don't's of FEA 531

9.3 Current Developments	542
References	544
Exercises	545
APPENDIX 1. CARTESIAN TENSOR ANALYSIS	553
APPENDIX 2. METHODS IN BEAM THEORY	561
APPENDIX 3. LAPLACE TRANSFORMS	566
APPENDIX 4. STRESS INTENSITY FACTORS FOR A FEW CASES	574
INDEX	585



# Preface

This book is a product of the understanding I developed of stress analysis applied to plastics, while at work at L. J. Broutman and Associates (LJBA) and as a lecturer in the seminars on this topic co-sponsored by LJBA and Society of Plastics Engineers. I believe that by its extent and level of treatment, this book would serve as an easy-to-read desktop reference for professionals, as well as a text book at the junior or senior level in undergraduate programs.

The main theme of this book is what to do with computed stress. To approach the theme effectively, I have taken the “stress category approach” to stress analysis. Such an approach is being successfully used in the nuclear power field. In plastics, this approach helps in the prediction of long term behavior of structures. To maintain interest I have limited derivations and proofs to a minimum, and provided them, if at all, as flow charts. In this way, I believe that one can see better the connection between the variables, assumptions, and mathematics.

I intend this book as a guide for the mechanical engineer working in plastics product design. Therefore, the use, rather than the description of mechanical behavior is treated preferentially. A working knowledge of the strength of materials, differential calculus, and ordinary differential equations should be adequate for most of the text, except that some knowledge of Laplace transforms may be helpful for Chapter 5. A cursory review of this topic is provided in Appendix 3. Wherever appropriate, I have provided interpretive comments, and have explained the use of existing material test practices based on the mechanical behavior of the materials. Examples in this book perform two major tasks. The first one is to illustrate a certain principle, and the second is to provide commentaries

about the differences between theory and practice. It is my hope that the student engineer will find these commentaries valuable.

Chapter 1 deals with the overall structure of the theory of elasticity, and in particular, with the fact that stresses caused by loads are basically different from stresses caused by displacements. Understanding this difference is fundamental to understanding plastics. It also introduces the stress category concept. Chapter 2 is an overview of how much can be done in plastics design by using existing techniques in metallics. The techniques used in this chapter range from simple press-fits to matrix structural analysis (using an example in piping flexibility). I should admit that in this chapter, the progression of complexity is rather fast. However, I was able to discuss a wider range of practical problems more compactly. Chapter 3 is basically about incipience of plastic flow. No flow rules are discussed. Chapter 4 returns to the stress category, and its relation to the failure modes known to occur in plastics. The concept of factor of safety is also discussed, as are the facts that it is a number having more details to it than meets the eye, and that rational product quality specifications decide what to look for in an analysis. Chapter 5 is a discussion of the application of viscoelasticity to static problems. I have presented a view that the so-called “pseudoelasticity” is a consequence of the correspondence principle. The examples in Laplace transforms are for illustration of the correspondence principle. If one were to assume that the Poisson’s ratio remains constant with time, and express all elastic properties in terms of  $E$  and  $\nu$  only, the Laplace transform approach will give results identical to those of pseudoelasticity. Thus, the Laplace transform has been used in this book as a pointer to pseudoelasticity technique. A proposal for ranking plastics is also included in this chapter. Chapter 6 concerns linear elastic fracture mechanics and applications. The basic four approaches to crack stability are discussed, but the application to the stress intensity approach is dealt with in detail because it permits computation. Calculation of the fatigue life of plastics is presented as a problem in the progressive increase of  $K_I$ . The crazing mechanism is explained using the background of stress intensity at crack tip. Chapter 7 discusses the micro- and macro-mechanics of fiber-reinforced plastics. A variety of results have been compiled in a manner that is suitable for computer coding. This chapter, with its exercises, may be suitable for a three-credit course.

Chapters 8 and 9 deal with finite elements. What do finite elements have to do with plastics? Finite elements are replacing classical methods of analysis in the same way that calculators replaced addition and multiplication tables. The design engineer invariably is a user of some finite element software. Covering an extensive topic such as this in one or even two chapters is presumptuous. Only the elementary ideas are discussed and I

hope the reader will gain enough confidence to refer to more detailed books on the subject. Chapter 8 discusses the steps involved in the finite element analysis, by taking a linear triangular element as the basic entity. Extensions to higher order elements, or more nodes per element are just indicated. A few exercises stand in for detailed discussion of the topic. Chapter 9 contains cautionary advice to any user of finite elements.

I have used a casual style in algebraic manipulations. At several places, I have used dots such as ( $\cdots$ ) to mean an exact repetition of the quantity in the previous step. This is not formal, but the reader who wants to get to the bottom line quickly will find this notation agreeable. In Chapter 5, I have used  $E'$  and  $E''$  with subscripts  $\sigma$  and  $\epsilon$ , to denote the manner in which the test data were obtained. This once again is nonstandard. I have used such shortcuts consciously with the hurried student in mind, the kind of student who might pick a formula from the text without pausing to look for the applicable restrictions. In Chapter 7, for example, the Tsai-Halpin formulas are summarized by the use of an unreadable symbol ( $\bullet$ ), a symbolism which, I believe, communicates the generality of the formulas.

Some curves in this text are reproductions taken from other texts, trade brochures or company manuals. I gratefully acknowledge the permissions from the organizations to reproduce them.

# Acknowledgments

The origins of this book are in the seminars I worked on with Dr. L. J. Broutman who co-sponsored them with the Society of Plastics Engineers. I gratefully acknowledge the support, resources, and encouragement I received from him, without which this book would not have been possible. Thanks are due to Prof. S. Kalpakjian and Dr. S. Nair, both of Illinois Institute of Technology, Chicago, and to my colleague Dr. Paul K. So, whose counsel and technical discussions were valuable to me all along the writing. I sincerely appreciate the advice on the style and grammar I received from my colleagues Mr. Ken E. Hofer, and Mr. Michael R. Roop. I thank the editors of Van Nostrand Reinhold, a little suggestion from whom sparked off this project. My wife Lalitha and sons Sriram and Parashar gave me loving support, cooperation, and understanding for the last three years of writing.