

**PROBLEMS IN
LASER PHYSICS**

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**G. Cerullo, S. Longhi, M. Nisoli,
S. Stagira, and O. Svelto**

*Politecnico di Milano
Milano, Italy*

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Preface

There is hardly any book that aims at solving problems typically encountered in the laser field, and this book intends to fill the void. Following some initial exercises related to general aspects in laser physics (Chapt. 1), the subsequent problems are organized along the following topics: (i) Interaction of radiation with matter either made of atoms or ions, weakly interacting with surrounding species, or made of more complicated elements such as molecules or semiconductors (Chapters 2 and 3). (ii) Wave propagation in optical media and optical resonators (Chapters 4 and 5). (iii) Optical and electrical pumping processes and systems (Chapter 6). (iv) Continuous wave and transient laser behaviors (Chapters 7 and 8). (v) Solid-state, dye, semiconductor, gas and X-ray lasers (Chapters 9 and 10). (vi) Properties of the output beam and beam transformation by amplification, frequency conversion and pulse compression or expansion (Chapters 11 and 12).

Problems are proposed here and solved following the contents of Orazio Svelto's *Principles of Lasers* (fourth edition; Plenum Press, New York, 1998). Whenever needed, equations and figures of the book mentioned above are currently used with an appropriate reference [e.g., Eq. (1.1.1) of the book is referred to as Eq. (1.1.1) of PL]. One can observe, however, that the types of problems proposed and discussed are of general validity and many of these problems have actually been suggested by our own long-time experience in performing theoretical and experimental researches in the field. Some of these problems are also directly related to real-world lasers (i.e., lasers, laser components and laser systems commonly found in research laboratories or commercially available). Therefore, the reader should be able to solve most of these problems even if his knowledge in laser physics has been acquired through studying other textbooks.

In each chapter, problems are first proposed all together and then solved at the end. This should encourage the reader to solve the problem by himself without immediately looking at the solution. Three kinds of problems are considered with attention being paid to a good balance between them:

1. Problems where one just needs to insert appropriate numbers into some important equation already provided in the previously mentioned book (*applicative problems*): they should help students to become more acquainted with important equations in laser physics and with the typical values of the corresponding parameters that are involved.
2. Problems where students are asked to prove some relevant equation left unproven

in the textbook (*demonstrative problems*): their purpose is to test the maturity acquired by demonstrating some, generally simple, passages.

3. Problems where students are asked to develop topics which go beyond those covered in the above book as well as in many other textbooks in the field (*evolutional problems*): their purpose is to increase the depth of knowledge in the laser field. Whenever appropriate, some hints for the solutions are also added, particularly for some more advanced demonstrative or evolutional problems. However, when the level of difficulty is deemed to be particularly high, a warning in the form “level of difficulty higher than average” is added at the end of the corresponding problem. This should help the reader, on the one hand, know when to apply himself harder and, on the other hand, not to get discouraged at a possible failure. Reading the solution should allow students to considerably enrich their basic knowledge in the field.

Lastly, care has been taken not to have the solution of one problem be dependent on the solution of a preceding problem in the chapter. This should allow more freedom for tackling problems not necessarily in sequential order.

Given the number of problems proposed and their wide variety, it is believed that a proficient student, upon solving these problems, should become more than well prepared to begin a research activity in laser physics and engineering as well as in the general field of photonics.

Giulio Cerullo
Stefano Longhi
Mauro Nisoli
Salvatore Stagira
Orazio Svelto

Milan, February 2001

Contents

	P	A
1. Introductory Concepts	1	
1.1. Spectrum of laser emission	1	5
1.2. Spectrum of visible light	1	5
1.3. Energy of a photon	1	5
1.4. Thermal energy	1	6
1.5. Population under thermal equilibrium of two levels	2	6
1.6. Small-signal gain of a ruby laser amplifier	2	7
1.7. Threshold inversion of a laser cavity	2	7
1.8. Temporal evolution of the population densities in a three-level system	2	8
1.9. Brightness of a diffraction limited beam	3	10
1.10. Comparison between the brightness of a lamp and that of an argon laser	3	11
1.11. Intensity on the retina of the sun light and of a He-Ne laser beam	3	11
1.12. Power spectrum of a wave-train of finite duration	4	12
1.13. Coherence time and coherence length of filtered light	4	13
1.14. Radiation pressure of a laser beam	4	14
1.15. Radiation pressure	4	14
2. Interaction of Radiation with Atoms and Ions	17	
2.1. Intensity and energy density of a plane em wave	17	23
2.2. Photon flux of a plane monochromatic wave	17	24
2.3. Number of modes of a blackbody cavity	17	24
2.4. Wien's law	17	26

	P	A
2.5. Blackbody cavity filled with a dispersive medium	18	27
2.6. Power irradiated by a blackbody emitter	18	28
2.7. Average mode energy	18	29
2.8. Spontaneous and stimulated emission rates	18	30
2.9. Natural broadening	18	30
2.10. Doppler broadening	19	33
2.11. Temperature of a blackbody with the same energy density of a He-Ne laser	19	33
2.12. Spontaneous lifetime and cross section	19	34
2.13. Radiative lifetime and quantum yield of the ruby laser transition	19	35
2.14. Radiative lifetime of the strongest transition of the Nd:YAG laser	20	35
2.15. Transient response of a two-level system to an applied signal	20	36
2.16. Gain saturation intensity	20	39
2.17. Population inversion of a homogeneously broadened laser transition	21	41
2.18. Strongly coupled levels	21	42
2.19. Amplification of a monochromatic em wave	21	43
2.20. Amplified Spontaneous Emission in a Nd:YAG rod	22	44
2.21. Saturated absorption coefficient	22	45
2.22. Peak absorption coefficient and linewidth	22	45
3. Energy Levels, Radiative, and Nonradiative Transitions in Molecules and Semiconductors	47	
3.1. Vibrational frequency of a diatomic molecule	47	53
3.2. Calculation of the elastic constant of a molecule.....	47	54
3.3. From the potential energy to the vibrational frequency.....	47	54
3.4. The Morse potential energy	47	55
3.5. Calculation of the Franck-Condon factor	48	56
3.6. Rotational constant of a diatomic molecule	48	57
3.7. Far-infrared absorption spectrum of an HCl molecule	49	58
3.8. The most heavily populated rotational level	49	59
3.9. The emission lines of a CO ₂ molecule	49	60
3.10. The law of mass action.....	49	60
3.11. Energies of the quasi-Fermi levels.....	50	62
3.12. The quasi-Fermi levels in GaAs	50	63
3.13. Derivation of the Bernard-Duraffourg condition	50	63
3.14. Laser levels in a semiconductor	50	64

	P	A
3.15. Frequency dependence of the gain of an inverted semiconductor.....	50	65
3.16. Gain calculation in a GaAs amplifier	51	67
3.17. Differential gain of a GaAs amplifier	51	68
3.18. Thickness of a quantum well: an order of magnitude estimate.....	51	69
3.19. An ideal quantum well.....	51	70
3.20. Energies of the quasi-Fermi levels in a semiconductor quantum well	52	71
3.21. Calculation of the gain bandwidth in a GaAs quantum well	52	73
4. Ray and Wave Propagation through Optical Media.....	75	
4.1. ABCD matrix of a spherical dielectric interface	75	81
4.2. ABCD matrix of a thin lens	75	83
4.3. ABCD matrix of a piece of glass.....	75	83
4.4. Reflection at a plane interface	75	84
4.5. An high reflectivity dielectric mirror	76	86
4.6. A Fabry-Perot interferometer	76	87
4.7. A scanning Fabry-Perot interferometer	76	88
4.8. An imaging optical system	76	88
4.9. The ABCD law for gaussian beams	77	89
4.10. A collimating lens	77	92
4.11. A simple optical processing system	77	93
4.12. A laser driller	78	94
4.13. An earth to moon laser rangefinder	78	95
4.14. An He-Ne laser	78	95
4.15. An Argon laser.....	78	96
4.16. Gaussian beam propagation through an optical system.....	79	97
4.17. Power conservation for a gaussian beam	79	99
4.18. A “soft” or gaussian aperture	79	100
4.19. A waist imaging system.....	79	101
4.20. Gaussian beam transformation by a lens.....	79	102
4.21. Focusing a gaussian beam inside a piece of glass	80	103
5. Passive Optical Resonators	105	
5.1. Stability of a resonator with concave mirrors	105	112
5.2. A concave-convex resonator	105	113
5.3. A simple two-mirror resonator.....	105	113

	P	A
5.4. Number of longitudinal modes in a resonator	105	113
5.5. Resonators for an Argon laser	106	114
5.6. A resonator for a CO ₂ laser	106	115
5.7. A near-planar resonator	106	116
5.8. Single-mode selection in a He-Ne laser	106	117
5.9. Spot sizes on the mirrors of a stable resonator	107	117
5.10. A plano-concave resonator	107	118
5.11. A near-concentric resonator	107	120
5.12. The unlucky graduate student	107	121
5.13. Resonator with an intracavity lens	108	122
5.14. Resonator for a cw-pumped Nd:YAG laser	108	124
5.15. Resonator for a Ti:sapphire laser	108	124
5.16. Location of the beam waist in a stable resonator	109	127
5.17. Properties of a symmetric confocal resonator	109	128
5.18. Asymmetric confocal resonators.....	109	129
5.19. A confocal unstable resonator.....	110	130
5.20. Unstable resonator with gaussian mirrors: properties of the output beam	110	131
5.21. Designing a gaussian mirror for an unstable resonator	110	132
5.22. Unstable resonator with a supergaussian mirror	110	133
6. Pumping Processes	135	
6.1. Critical pump rate in a lamp-pumped Nd:YLF laser.....	135	141
6.2. Pump rate expression for longitudinal pumping	135	142
6.3. Laser spot size in a longitudinally pumped Ti:Al ₂ O ₃ laser under optimum pumping conditions.....	135	143
6.4. Optical pumping of a Ti:Al ₂ O ₃ laser: a design problem.....	136	143
6.5. Doping in a solid-state laser medium.....	136	144
6.6. A transversely pumped high-power Nd:YAG laser.....	136	145
6.7. Longitudinal vs. transverse pumping in Nd:YAG laser.....	136	145
6.8. Threshold power in a double-end pumped Nd:YVO ₄ laser...	137	146
6.9. Threshold power in a quasi-three level laser: the Yb:YAG case.....	137	146
6.10. Threshold pump power of a Nd:glass fiber laser.....	137	147
6.11. Pump absorption in a Nd:glass fiber laser.....	138	147
6.12. Maximum output intensity in a Nd:glass amplifier.....	138	149
6.13. Electron temperature in a Boltzmann distribution.....	138	149
6.14. How to reduce the size of a He-Ne laser tube?.....	138	150
6.15. Thermal and drift velocities of electrons in a He-Ne laser...	139	150
6.16. A He-Ne laser: pump rate vs. pump current.....	139	151

	P	A
6.17. Scaling laws and performances in longitudinally pumped gas lasers.....	139	152
6.18. Pump rate vs. pumping current in Ar ⁺ lasers.....	139	152
6.19. Ar ⁺ lasers: pump efficiency vs. pump power	140	153
7. Continuous Wave Laser Behavior	155	
7.1. Calculation of logarithmic loss.....	155	163
7.2. Calculation of cavity photon lifetime.....	155	163
7.3. Four-level laser with finite lifetime of the lower laser level..	155	164
7.4. Rate equations analysis of a three-level laser.....	156	165
7.5. Threshold condition in a ruby laser.....	156	167
7.6. Thermal lensing in a microchip Nd:YAG laser.....	156	167
7.7. Transverse efficiency in an end-pumped four-level laser	157	168
7.8. Threshold and slope-efficiency calculations in a longitudinally-pumped Nd:YAG laser	157	169
7.9. Estimate of internal laser losses.	157	171
7.10. Calculation of optimum output coupling.....	158	171
7.11. Longitudinal efficiency in a standing-wave laser.	158	172
7.12. Dispersion relation for a Lorentzian line.....	158	175
7.13. Frequency pulling in a homogeneously-broadened laser.....	159	178
7.14. Calculation of frequency pulling in a He-Xe laser.....	159	180
7.15. Quantum limit to the laser linewidth.....	159	180
7.16. Tuning of a Ti:sapphire laser by a birefringent filter.....	159	182
7.17. Transverse mode selection.....	160	183
7.18. Single longitudinal mode oscillation in an inhomogeneously broadened laser.....	160	184
7.19. Suppression of spatial hole burning by the twisted-mode technique.....	160	184
7.20. Single-longitudinal mode selection by an intracavity etalon.....	161	185
8. Transient Laser Behavior	187	
8.1. Relaxation oscillations in a Nd:YAG laser.....	187	193
8.2. Noise spectrum of the output power for a four-level laser...	187	193
8.3. Fast Q-switching in a Nd:YLF laser.....	187	195
8.4. Calculation of the pulse energy and pulse duration in a repetitively Q-switched Nd:YAG laser.....	188	196
8.5. Quarter-wave voltage in a Q-switch Pockels cell.....	188	197
8.6. Active Q-switching in a three-level laser.....	188	198
8.7. Calculation of the beam deflection angle by an acousto-optic modulator.....	189	200

	P	A
8.8. Mode-locking of sidebands modes with random amplitudes.....	189	201
8.9. Chirped Gaussian pulses with quadratic phase locking relations.....	189	202
8.10. On the periodicity of mode-locked signals.....	189	203
8.11. Phase locking condition for second-harmonic mode-locking.....	190	204
8.12. Pulsewidth calculation in an actively mode-locked Nd:YAG laser.....	190	205
8.13. Gaussian pulse analysis of frequency mode locking.....	190	206
8.14. Mode-locking in a He-Ne laser.....	190	208
8.15. Harmonic mode-locking of a laser in a linear cavity.....	191	209
8.16. Calculation of pulse energy and peak power in a passively mode-locked Nd:YAG laser.....	191	210
8.17. Pulse duration in an idealized Kerr lens mode-locked Ti:Sapphire laser.....	191	210
8.18. Pulse duration in a soliton-type Ti:sapphire mode-locked laser.....	192	212
8.19. Pulse broadening in a quartz plate.....	192	212
8.20. Self-imaging of a mode-locked pulse train.....	192	213
9. Solid-State, Dye, and Semiconductor Lasers.....	215	
9.1. Slope efficiency in a Ti:Al ₂ O ₃ laser.....	215	221
9.2. Output power from a Nd:YAG laser.....	215	221
9.3. A Nd:YVO ₄ laser in the fog.....	215	222
9.4. A green solid-state laser.....	216	222
9.5. Yb:YAG laser vs. Nd:YAG laser.....	216	224
9.6. Anisotropy in a Cr:LiSAF laser rod.....	217	225
9.7. Threshold pump power in longitudinal pumping: ground and excited states contribution.....	217	227
9.8. Threshold pump power in a dye laser: triplet-triplet contribution.....	218	228
9.9. Slope efficiency in a dye laser.....	218	230
9.10. A laser cascade.....	218	232
9.11. Longitudinal modes in a semiconductor laser.....	218	232
9.12. Beam astigmatism in a semiconductor laser.....	219	233
9.13. Current threshold in a GaAs/AlGaAs laser.....	219	234
9.14. Slope efficiency in a GaAs/AlGaAs laser.....	219	235
9.15. Distributed feedback in a semiconductor laser.....	220	236
9.16. Current threshold in a quantum-well laser.....	220	237
9.17. Carrier density in a VCSEL at threshold.....	220	237

	P	A
10. Gas, Chemical, Free-Electron, and X-Ray Lasers	239	
10.1. Low-density laser emitting in the infrared.....	239	243
10.2. Low-density laser emitting in the UV - soft X region.....	239	243
10.3. High-power lasers for material processing.....	239	244
10.4. Internal structure of He-Ne lasers.....	239	244
10.5. Maximum output power in He-Ne lasers.....	240	245
10.6. Internal structure of high-power Ar ⁺ lasers.....	240	246
10.7. Output vs. pump power in Ar ⁺ lasers.....	240	246
10.8. Current density in a low power CO ₂ laser.....	240	247
10.9. Voltage drop in a low power CO ₂ laser tube.....	240	248
10.10. Rotational transitions in a CO ₂ laser.....	241	248
10.11. Mode locking of a CO ₂ laser.....	241	249
10.12. ASE threshold for a N ₂ laser.....	241	250
10.13. Pump power in a KrF excimer laser at threshold.....	242	251
10.14. Cold reaction in a HF chemical laser.....	242	252
10.15. Transition linewidths in the soft-X-ray spectral region.....	242	252
10.16. A free-electron laser operating in the soft-X-ray region.....	242	253
11. Properties of Laser Beams	255	
11.1. Complex degree of coherence for a quasi monochromatic wave	255	261
11.2. Measurement of the spatial coherence by a Young interferometer	255	262
11.3. Destroy of spatial coherence by rotation of a ground glass... ..	255	262
11.4. Comparison of temporal coherence between a thermal source and a laser.....	256	263
11.5. Temporal coherence of white light.....	256	264
11.6. Relation between first-order degree of temporal coherence and fringe visibility in a Michelson interferometer.....	256	264
11.7. Degree of temporal coherence for a low-pressure discharge lamp	256	266
11.8. Temporal coherence of a gas laser oscillating on N axial modes	257	268
11.9. An interference experiment with partially coherent light.....	257	269
11.10. Spatial coherence of the light from the sun.....	257	269
11.11. An astronomic calculation based on spatial coherence of stellar radiation.....	258	270
11.12. Beam divergence of a partially-coherent laser beam.....	258	270
11.13. Focusing of a perfectly-coherent spatial beam.....	258	271
11.14. M^2 factor of a Nd:YAG laser	258	272

	P	A
11.15. Brightness of a high-power CO ₂ laser	259	272
11.16. Grain size of the speckle pattern as observed on a screen....	259	273
11.17. Grain size of the speckle pattern as seen by a human observer.....	259	274
11.18. Correlation function and power spectrum of a single- longitudinal mode laser.....	259	274
12. Laser Beam Transformation: Propagation, Amplification, Frequency Conversion, Pulse Compression, and Pulse Expansion		277
12.1. Propagation of a multimode beam	277	285
12.2. Amplification of long pulses by a Nd:YAG amplifier	277	286
12.3. Amplification of short pulses by a Nd:YAG amplifier	277	286
12.4. Extraction efficiency of a two-pass amplifier	278	287
12.5. Saturation fluence in a quasi-three-level amplifier	278	289
12.6. Maximum output fluence from an amplifier with losses	278	290
12.7. Theoretical limit to the maximum intensity of an amplifier ..	279	291
12.8. Index of refraction of an extraordinary wave in a uniaxial crystal	279	292
12.9. Double refraction in a uniaxial crystal	279	293
12.10. Second harmonic conversion of a Ti:sapphire laser in a BBO crystal	279	294
12.11. Second harmonic conversion efficiency in a KDP crystal ...	280	296
12.12. Second harmonic generation with a Gaussian beam	280	297
12.13. Frequency doubling of a Gaussian beam in a KDP crystal ..	281	298
12.14. Effective nonlinear coefficient of a KDP crystal	281	299
12.15. Threshold pump intensity of an optical parametric oscillator	282	301
12.16. Collinear parametric generation in a BBO crystal	282	301
12.17. Noncollinear parametric generation in a BBO crystal	282	302
12.18. Nonlinear index n_2 of sapphire	283	305
12.19. Pulse spectral broadening due to self-phase modulation in a Kerr medium	283	305
12.20. Spectral broadening of a 20-fs pulse in a hollow fiber filled with argon	283	306
12.21. Group delay dispersion of a medium	284	307
12.22. Dispersion-induced broadening of a 10-fs pulse in a fused silica plate	284	308

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