

International Series in Operations Research & Management Science

Volume 204

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Elaine Chew

Mathematical and Computational Modeling of Tonality

Theory and Applications

 Springer

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ISSN 0884-8289 ISSN 2214-7934 (electronic)
ISBN 978-1-4614-9474-4 ISBN 978-1-4614-9475-1 (eBook)
DOI 10.1007/978-1-4614-9475-1
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2013953210

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Printed on acid-free paper

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This book is dedicated to

*my father, Chew Kim Lin, who instilled in me
a love for mathematics,*

*George Dantzig, in memoriam, who gave me
my first glimpse into research, and*

*Jeanne Bamberger, who showed me I could
combine it all with music*

Preface

Blending ideas from operations research, music psychology, music theory, and cognitive science, this book aims to tell a coherent story of how tonality pervades our experience, and hence our models, of music.

The story is told through the developmental stages of the Spiral Array model for tonality, a geometric model designed to incorporate and represent principles of tonal cognition, thereby lending itself to practical applications of tonal recognition, segmentation, and visualization. Mathematically speaking, the coils that make up the Spiral Array model are in effect helices, a spiral referring to a curve emanating from a central point. The use of “spiral” here is inspired by spiral staircases, intertwined spiral staircases: nested double helices within an outer spiral.

The book serves as a compilation of knowledge about the Spiral Array model and its applications, and is written for a broad audience ranging from the layperson interested in music, mathematics, and computing to the music scientist–engineer interested in computational approaches to music representation and analysis, from the music–mathematical and computational sciences student interested in learning about tonality from a formal modeling standpoint to the computer musician interested in applying these technologies in interactive composition and performance. Some chapters assume no musical or technical knowledge, and some are more musically or computationally involved.

I am extremely pleased that this book is to appear fifteen years after the eureka moment that gave rise to the Spiral Array model, and five years—and five house moves, including one cross-country and one cross-Atlantic—after the book proposal, formulated and accepted while I held the Edward, Frances, and Shirley B. Daniels Fellowship at the Radcliffe Institute for Advanced Study. The collaborators who have contributed to this volume include Alexandre R. J. François, Ching-Hua Chuan, and Yun-Ching Chen; our joint work forms the basis of the chapters on visualization, audio key finding, and pitch spelling. Alex is additionally author of the MuSA_RT Mac App, an interactive visualization software based on the Spiral Array that is a part of the supplemental material for this book.

This compendium would not have been possible without the support of Jeanne Bamberger, who has been a mentor well beyond my doctoral research, Kim Lin Chew, my father and the only person I know willing to proofread equations, and other long-suffering members of my family. I thank Jordan Smith for his last-minute voluntary proofreading, Doug Keislar for his (in)voluntary edits to the

book draft, the late Lindy Hess for her generous advice on the book proposal, Matthew Amboy for speedy feedback on the book drafts, Camille Price, incoming series editor, for her steadfast encouragement over the years, and Fred Hillier, a former teacher, whose optimism and impending departure as Series Editor provided the catalyst to finally complete the book.

Last but not least, I thank the anonymous student who asked the seemingly innocuous question, “What do you mean by key?,” that started this whole undertaking.

London, Singapore, Los Angeles, Boston, August 2013

Elaine Chew

Contents

Part I Introduction

1	Tonality	3
1.1	What is Tonality?	5
1.2	Elusive Yet Pervasive	6
1.3	Inception	8
1.4	Design Principles	10
1.5	Straddling Cultures	12
1.6	Overview	14
	References	17
2	An Abbreviated Survey	19
2.1	Spiral Array Overview	20
2.1.1	Tonality in a Nutshell	20
2.1.2	The Spiral Array	20
2.1.3	The Center of Effect	22
2.2	Spatial Models for Pitch Relations	24
2.2.1	Cognitive Representations of Musical Pitch	24
2.2.2	The Tonnetz	26
2.2.3	Isomorphic Representations	27
2.2.4	Transformational Theory	28
2.3	An Early Interior Point Algorithm	30
2.3.1	von Neumann’s Center of Gravity Algorithm	32
2.3.2	Dantzig’s Bracketing Technique	33
	References	36

Part II The Model

3	The Spiral Array	41
3.1	The Pitch Class Spiral	42
3.1.1	Description 1: A Derivation of the Harmonic Network	42
3.1.2	Description 2: A Closer Look	43

- 3.1.3 Discussion 1: How do the Intervals Add Up? 44
- 3.1.4 Discussion 2: The Inherent Toroid Structure 45
- 3.1.5 Mathematical Representation 45
- 3.2 Intervals in the Spiral Array. 46
 - 3.2.1 Relations Inherited from the Harmonic Network 47
 - 3.2.2 Interval Distances. 47
- 3.3 Representing Chords 48
 - 3.3.1 Description: Center of Triangle 48
 - 3.3.2 Example 1: The C Major Chord Representation 50
 - 3.3.3 Example 2: The A Minor Chord Representation. 50
 - 3.3.4 Mathematical Representation 51
 - 3.3.5 Spatial Geometry 51
- 3.4 Key Representations 53
 - 3.4.1 The Major Key Representation. 53
 - 3.4.2 Mathematical Representation 54
 - 3.4.3 The Minor Key Representation 54
 - 3.4.4 Spatial Geometry 56
- 3.5 An Array of Spirals: A Discussion 57
- 3.6 Summary of Definitions 59
- References 60

Part III Key Finding

- 4 The CEG Algorithm (Part I).** 63
 - 4.1 Introduction to the CEG Key Finding Method 64
 - 4.1.1 The Center of Effect. 64
 - 4.1.2 Why the Algorithm Works 64
 - 4.2 Key-Finding Example Using “Simple Gifts” 65
 - 4.2.1 The Melody 66
 - 4.2.2 Model Parameters. 66
 - 4.2.3 The Center of Effect Generator 67
 - 4.2.4 Output Keys and Distances 67
 - 4.2.5 Step-by-Step Geometry 67
 - References 71
- 5 The CEG Algorithm (Part II): Validation** 73
 - 5.1 Model Validation 73
 - 5.1.1 Test Data. 74
 - 5.2 Comparing Key-Finding Algorithms 75
 - 5.2.1 Method 3: The Shape Matching Algorithm 75
 - 5.2.2 Method 2: The Probe Tone Profile Method 76
 - 5.2.3 Method 1: The CEG Algorithm 77

- 5.3 Detailed Analysis of Comparison Results 79
 - 5.3.1 Results Summary 79
 - 5.3.2 Case 1: Same Performance by All Three Algorithms 80
 - 5.3.3 Case 1a: Almost Same Performance by All Three Algorithms 81
 - 5.3.4 Case 2: SMA Worse than CEG and PTPM 82
 - 5.3.5 Case 3a: Worst Case for the CEG Algorithm. 84
 - 5.3.6 Case 3b: Best Cases for the CEG Algorithm 84
 - 5.3.7 Case 4a: Worst Cases for the PTPM Algorithm 86
 - 5.3.8 Case 4b: Best Cases for the PTPM Algorithm 87
- 5.4 Discussion: Strengths and Weaknesses 88
- 5.5 Evaluation Method Discussion 90
- References 91

Part IV Segmentation

- 6 Determining Key Boundaries 95**
 - 6.1 A Mathematical Model for Tonality 97
 - 6.1.1 Placement of Pitch Classes, Chords and Keys 98
 - 6.2 An Algorithm for Finding Key-Change Boundaries 100
 - 6.2.1 The Center of Effect and Distance to Key. 100
 - 6.2.2 Problem Statement and Algorithm 101
 - 6.3 Two Examples 102
 - 6.3.1 Example 1: Minuet in G by Bach. 103
 - 6.3.2 Example 2: March in D by Bach 104
 - 6.4 Conclusions 106
 - References 107
- 7 Argus Segmentation Method 109**
 - 7.1 Introduction 109
 - 7.2 The Spiral Array Model 111
 - 7.2.1 The Model’s Structure. 111
 - 7.2.2 Parameter Selection 113
 - 7.2.3 The Center of Effect. 114
 - 7.2.4 The Argus Algorithm 114
 - 7.3 Messiaen’s Regard IV. 116
 - 7.3.1 Manual Segmentation 116
 - 7.3.2 Automatic Segmentation 118
 - 7.4 Messiaen’s Regard XVI. 122
 - 7.4.1 Manual Segmentation 122
 - 7.4.2 Automatic Segmentation 122

- 7.5 Discussion: Why Does It Work? 125
 - 7.5.1 Geometric Shapes Corresponding to Pitch Sets
in Messiaen’s Examples 125
 - 7.5.2 What are the Structural Alternatives? 126
- 7.6 Conclusions 129
- References 130

Part V Pitch Spelling

- 8 Real-Time Pitch Spelling 133**
 - 8.1 Why Spell? 133
 - 8.2 Related Work. 136
 - 8.3 The Algorithm 139
 - 8.3.1 The Spiral Array Model 140
 - 8.3.2 Pitch-name Assignment 142
 - 8.3.3 Comparison with Other Pitch-spelling Models 144
 - 8.3.4 Segmentation Strategy. 145
 - 8.4 Computational Results. 147
 - 8.4.1 Brief Review of Prior Experiments. 150
 - 8.4.2 Approaching the Bootstrapping Algorithm. 150
 - 8.4.3 Bootstrapping Algorithm Experiments. 152
 - 8.4.4 Tests on the *Song of Ali-Shan* 153
 - 8.5 Conclusions 154
 - References 156

Part VI Visualization

- 9 MuSA.RT. 159**
 - 9.1 Introduction 159
 - 9.2 The Spiral Array Model for Tonality 162
 - 9.2.1 The Spiral Array Model 162
 - 9.2.2 The Center of Effect. 163
 - 9.2.3 Computational Algorithms. 164
 - 9.3 The MuSA.RT Opus 2 System. 165
 - 9.3.1 Software Architecture for Immersipresence 165
 - 9.3.2 MuSA.RT System Architecture 166
 - 9.3.3 Implementation 167
 - 9.3.4 Visual Display 167
 - 9.3.5 3D Rendering. 168
 - 9.3.6 View Manipulation 168

- 9.4 Case Studies 169
 - 9.4.1 Case I. Pachelbel’s Canon in D 169
 - 9.4.2 Case II. Prelude from Bach’s Cello Suite No. 1 171
 - 9.4.3 Case III. Barber’s Adagio for Strings Op. 11 173
- 9.5 Discussion and Conclusions 177
- References 177

- 10 Visible Humor 179**
 - 10.1 MuSA.RT and Visualization 180
 - 10.1.1 Seeing Style Differences 181
 - 10.2 Expectations Violated 182
 - 10.2.1 The Jazz Ending 182
 - 10.2.2 Improbable Harmonies 184
 - 10.2.3 Excessive Repetition 186
 - 10.3 Remarks on Humor and Tonality 187
 - References 189

Part VII Extensions

- 11 Sensitivity Analysis 193**
 - 11.1 Introduction 194
 - 11.1.1 Related Work 195
 - 11.2 System Description 196
 - 11.3 Basic System 196
 - 11.3.1 Fuzzy Analysis Technique 199
 - 11.3.2 Spiral Array Model and the Center
of Effect Algorithm 202
 - 11.3.3 Key Determination: Relative Distance Policy 203
 - 11.3.4 Key Determination: Average Distance Policy 204
 - 11.4 Evaluation of the Basic System 204
 - 11.4.1 Overall Results Over Time 205
 - 11.4.2 When Audio Outperforms Symbolic Key-finding 206
 - 11.4.3 Results of Global Key Across Periods 210
 - 11.5 System Extensions 210
 - 11.5.1 Modified Spiral Array with Piano Signals 211
 - 11.5.2 Fundamental Frequency Identification 212
 - 11.5.3 Post-weight Balancing 214
 - 11.6 In-depth Analysis of Results 215
 - 11.7 Conclusions and Discussion 220
 - References 221

Appendix A: Model Calibration 223

Appendix B: CEG Key Finding 261

Appendix C: Glossary 295

Author Index 299

Subject Index 301