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# Topics in Matroid Theory

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*To Lenio, Vasilina, and Alexios*

# Preface

The purpose of this monograph is to expose a less-known decomposition result in matroid theory that provides a structural characterization of graphic matroids, and show how this can be extended to signed-graphic matroids. The immediate algorithmic consequences of the decomposition are also examined. In order to make the exposition self-contained we also provide a brief, but nevertheless solid introduction to the elements of matroid theory, by presenting the way it exhibits itself in three different contexts, namely graph theory, vector spaces, and transversal theory. This book is intended for graduate students and researchers from graph theory, operations research, and combinatorial optimization, who are interested in theoretical and algorithmic applications of matroid theory.

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Thessaloniki, Greece, March 2013 Leonidas S. Pitsoulis

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# Symbols

| | |
|---|---|
| $(E, \mathscr{I})$ | Independence system |
| $\alpha(G)$ | Edge connectivity number of a graph $G$ |
| $\cap_i M_i$ | Intersection of matroids |
| $\kappa(G)$ | Vertex connectivity number of a graph $G$ |
| $\kappa(M)$ | Vertical connectivity number of a matroid $M$ |
| $\lambda(G)$ | Connectivity number of a graph $G$ |
| $\lambda(M)$ | Connectivity number of a matroid $M$ |
| $\langle X \rangle$ | Span of a set of vectors $X$ |
| $\mathbb{N}$ | Natural numbers |
| $\mathbb{R}$ | Real numbers |
| $\mathbb{Z}$ | Integer numbers |
| $\mathbb{Z}_+$ | Non-negative integer numbers |
| $\mathscr{B}$ | Family of bases |
| $\mathscr{C}$ | Family of circuits |
| $\mathscr{I}$ | Family of independent sets |
| $\pi(M,B,Y)$ | Partition of $Y$ as determined by $B$ |
| $\Sigma(G,\sigma)$ | Signed graph $\Sigma$ with underlying graph $G$ and sign function $\sigma$ |
| $\Sigma/\{e\}$ | Contraction of $e \in E(\Sigma)$ in signed graph $\Sigma$ |
| $\Sigma\backslash\{e\}$ | Deletion of $e \in E(\Sigma)$ in signed graph $\Sigma$ |
| $\dim(V)$ | Dimension of vector space $V$ |
| $\mathbf{e}_k$ | Unit vector |
| $A/Y$ | Contraction of $Y \in columns(A)$ in matrix $A$ |
| $A \Delta B$ | Symmetric difference of sets |
| $A(:, j)$ | $j$-th Column of matrix $A$ |
| $A(i,:)$ | $i$-th Row of matrix $A$ |
| $A-B$ | Difference of sets |
| $A^T$ | Transpose of matrix $A$ |
| $A_{\overrightarrow{\Sigma}}$ | Incidence matrix of a bidirected graph $\overrightarrow{\Sigma}$ |
| $A_{\overrightarrow{G}}$ | Incidence matrix of a directed graph $\overrightarrow{G}$ |
| $A_\Sigma$ | Incidence matrix of a signed graph $\Sigma$ |
| $A_G$ | Incidence matrix of a graph $G$ |
| $C(B,v)$ | Components determined by bridges |
| $cl(X)$ | Closure of a set $X$ |

| | |
|---|---|
| $columns(A)$ | Column indices of matrix $A$ |
| $d_G(v)$ | Degree of a vertex $v \in V(G)$ |
| $G \cap H$ | Intersection of graphs $G$ and $H$ |
| $G/Y$ | Contraction of $Y \subseteq E(G)$ in graph $G$ |
| $G.Y$ | Contraction to $Y \subseteq E(G)$ in graph $G$ |
| $G \cup H$ | Union of graphs $G$ and $H$ |
| $G \backslash X$ | Deletion of $X \subseteq E(G)$ in graph $G$ |
| $G \vert X$ | Deletion to $X \subseteq E(G)$ in graph $G$ |
| $GF(2)$ | Binary field |
| $GF(3)$ | Ternary field |
| $I_n$ | Identity matrix of size $n$ |
| $k_G$ | Number of connected components of a graph $G$ |
| $K_{n_1,n_2,\ldots,n_k}$ | Complete $k$-partite graph |
| $K_n$ | Complete graph on $n$ vertices |
| $lr(X)$ | Low rank of a set $X$ |
| $M/X$ | Contraction of elements $X$ from matroid $M$ |
| $M.X$ | Contraction to elements $X$ in matroid $M$ |
| $M \backslash X$ | Deletion of $X \in columns(A)$ in matrix $A$ |
| $M \backslash X$ | Deletion of elements $X$ from matroid $M$ |
| $M \vert X$ | Deletion to elements $X$ in matroid $M$ |
| $M(E, \mathscr{F})$ | Transversal matroid of set system $(E, \mathscr{F})$ |
| $M(E, \mathscr{I})$ | Matroid $M$ on $E$ with independence family $\mathscr{I}$ |
| $M(G)$ | Graphic matroid of $G$ |
| $M[A]$ | Vector matroid of matrix $A$ |
| $M^*$ | Dual matroid of $M$ |
| $M_1 \cong M_2$ | Matroid $M_1$ isomorphic with $M_2$ |
| $N(A)$ | Nullspace of matrix $A$ |
| $N(A^T)$ | Row space of matrix $A$ |
| $N_G(v)$ | Neighborhood of $v$ in $G$ |
| $o$ | Orientation function of a graph $G$ |
| $R(A)$ | Column space of matrix $A$ |
| $r(G)$ | Rank of a graph $G$ |
| $r(X)$ | Rank of a set $X$ in independence systems |
| $r(X)$ | Rank of a set of vectors $X$ |
| $R_{G_1} \circledast_Y R_{G_2}$ | Star composition of $G_1$ and $G_2$ in $Y$ |
| $rows(A)$ | Row indices of matrix $A$ |
| $S^{\perp}$ | Orthogonal complement of subspace $S$ |
| $U_{k,n}$ | Uniform matroid on $n$ elements and rank $k$ |
| $Y(B,v)$ | Partition of a cocircuit |