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VOLUME 85

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Statistical Decision Problems

Selected Concepts and Portfolio Safeguard Case Studies

Springer

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ISSN 1931-6828
ISBN 978-1-4614-8470-7 ISBN 978-1-4614-8471-4 (eBook)
DOI 10.1007/978-1-4614-8471-4
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2013946847

Mathematics Subject Classification (2010): 62C05, 62J05, 62J12, 62P05, 62P30, 90B50, 90B90, 90C05, 90C11, 90C20, 90C26, 90C90, 91B06, 91B30, 91B70

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

Scope

In an abstract form, statistical decision making is an optimization problem that uses available statistical data as an input and optimizes an objective function of interest with respect to decision variables subject to certain constraints. Typically, uncertainty encoded in statistical data can be translated into five basic notions: *likelihood*, *entropy*, *error*, *deviation*, and *risk*. As a result, the majority of statistical decision problems can be tentatively divided into four major categories: (i) likelihood maximization, (ii) entropy maximization (relative entropy minimization), (iii) error minimization (regression), and (iv) decision models in which deviation or risk is either minimized or constrained. All these problems may include so-called technical constrains on decision variables, e.g., box and cardinality constraints. It is also common to optimize one of the corresponding five functionals while to constrain another, e.g., maximizing entropy subject to a constraint on deviation, or to find a trade-off between one of the functionals and the expected value of the quantity of interest. The book aims to demonstrate how to use these "building blocks": likelihood, entropy, error, deviation, and risk to formulate statistical decision problems arising in various risk management applications, e.g., optimal hedging, portfolio optimization, portfolio replication, cash flow matching, and classification and how to solve those problems in optimization package *Portfolio Safeguard* (PSG).

Content

The book consists of three parts: selected concepts of statistical decision theory (Part I), statistical decision problems (Part II), and case studies with PSG (Part III). Part I presents a general theory of error, deviation, and risk measures to be used in various statistical decision problems and also discusses probabilistic inequalities with deviation measures such as generalized Chebyshev's and Kolmogorov's inequalities. Part II covers five major topics: parametric and nonparametric estimation based on the maximum likelihood principle, entropy maximization problems, unconstrained and constrained linear regression with general error measures, classification with logistic regression and with support vector machines (SVMs), and statistical decision problems with general deviation and risk measures. Part III discusses 21 case studies of typical statistical decision problems arising in risk management, particularly in financial engineering, and demonstrates implementation of those problems in PSG. All case studies are closely related to theoretical Part II and are examples of statistical decision problems from the four categories (i)–(iv).

Audience

The book is aimed at practitioners working in the areas of risk management, decision making under uncertainty, and statistics. It can serve as a quick introduction into the theory of general error, deviation, and risk measures for the graduate students, engineers, and statisticians interested in modeling and managing risk in various applications such as optimal hedging, portfolio replication, portfolio optimization, cash flow matching, structuring of collateralized debt obligations (CDOs), classification, sparse signal reconstruction, and therapy treatment optimization, to mention just a few. It can also be used as a supplementary reading for a number of graduate courses including but not limited to those of statistical analysis, models of risk, data mining, stochastic programming, financial engineering, modern portfolio theory, and advanced engineering economy.

Optimization Software: Portfolio Safeguard

PSG is an advanced nonlinear mixed-integer optimization package for solving a wide range of optimization, statistics, and risk management problems. PSG is a product of American Optimal Decisions, Inc. (see www.aorda.com). Although PSG is a general-purpose decision support tool, the focus application areas are risk management, financial engineering, military, and medical applications. PSG is based on a simple but powerful idea: for every engineering area, identify most commonly used nonlinear functions and include them in the package as independent built-in objects. Each function is defined by a function type, parameters, and a matrix of data (e.g., scenario matrix or covariance matrix). Specialized algorithms, built for different types of functions, efficiently optimize large-scale nonlinear functions, such as probability, value-at-risk (VaR), and omega functions, which are typically beyond the scope of commercial packages. The built-in function library provides simple and convenient interface for evaluating functions and their derivatives, for constructing optimization problems and solving them, and for analyzing solutions. No programming experience is required to use PSG.

PSG operates in four programming environments: Shell (Windows), MATLAB, C++, and Run-File (Text). The standard PSG setup includes case studies from various areas with emphasis on financial engineering applications, such as portfolio optimization, asset allocation, selection of insurance, hedging with derivative contracts, bond matching, and structuring of CDOs.

PSG can be downloaded from the American Optimal Decisions web site: www.aorda.com/aod. Four types of licenses are available: Freeware Express, Regular, Academic, and Regular Business. Freeware Express Edition limits the number of decision variables per function to ten. The Regular PSG edition has a free 30-day trial. After installing PSG, case study projects can be viewed in the Case Studies folder in File tab of the PSG menu. In order to modify an existing case study, it should be copied into Work directory. Tutorials about PSG and case study descriptions can be found in Help tab of the menu.

Acknowledgments

We thank Dr. Bogdan Grechuk at the University of Leicester, UK, and Dr. Anton Molyboha at Teza Technologies for carefully proofreading the portions of the manuscript and for providing valuable comments. We are also grateful to Dr. Viktor Kuzmenko at American Optimal Decisions, Inc. for the technical support and assistance with running numerical experiments for the case studies. We express our deepest gratitude to our families for their constant encouragement, support, and patience.

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Contents

Nomenclature

$\sim$	"has distribution"
$[x]_+$	$\max\{x, 0\}$ for real-valued x
$\partial\mathscr{F}(X)$	subdifferential of convex functional $\mathscr{F}$ at X
$\succeq_{(n)}$	non-strict preference relation with respect to n^{th}-order stochastic dominance
$\\|X\\|_p$	p-norm of X in $\mathscr{L}^p$, $p \in [1, \infty]$
$\mathscr{A}$	acceptance set
$\text{AvgDD}(\xi)$	average drawdown of sample path ξ
β_i	beta of asset $i \in \{1, \ldots, n\}$ in the capital asset pricing model (CAPM)
$\mathscr{C}$	set of parameters $c_0, c_1, \ldots, c_n$
$\text{CDaR}_\alpha(\xi)$	conditional drawdown-at-risk (CDaR) of sample path ξ
$\text{Cov}(X, Y)$	covariance of X and Y
$\text{CVaR}_\alpha(X)$	conditional value-at-risk (CVaR) of X
$\text{CVaR}_\alpha^\Delta(X)$	conditional value-at-risk (CVaR) deviation of X
$\text{CVaR}_\alpha^-(L)$	conditional value-at-risk (CVaR) for loss L
$\mathscr{D}(X)$	deviation measure $\mathscr{D}$ of X
$D_{\text{KL}}(X\\|Y)$	Kullback–Leibler divergence measure (relative entropy) of X and Y
$\mathscr{E}(X)$	error measure $\mathscr{E}$ of X
$E[X]$	expected value of X
e	n-dimensional unit vector $(1, \ldots, 1)$
$F_X(t)$	cumulative distribution function (CDF) of X
$f_X(t)$	probability density function (PDF) of X
$F_X^{-1}(s)$	inverse of the cumulative distribution function (CDF) of X
$\Phi(t)$	cumulative distribution function (CDF) of the standard normal random variable
$\Gamma(x)$	Gamma function of real-valued x
$H_\alpha(X)$	Renyi entropy of X
$I_{\{\cdot\}}$	indicator function (1 if the condition in the curly brackets is true and 0 otherwise)
inf	essential infimum of X ($\text{ess}\inf X$)

$\mathscr{L}^p$	Lebesgue space of integrable functions, $p \in [1, \infty]$
Λ	covariance matrix
Λ^{-1}	inverse of covariance matrix Λ
$\mathscr{M}$	sigma-algebra over space of probability events
$M_X(t)$	moment generating function (MGF) of X
$\mathrm{MAD}(X)$	mean absolute deviation (MAD) of X
$\mathrm{MaxDD}(\xi)$	maximum drawdown of sample path ξ
$\mathrm{med}\,(X)$	median of X
μ	mean value of X
μ_M	expected rate of return of master fund (market portfolio)
$\mathbb{N}$	set of positive integer numbers
$N\left(\mu, \sigma^2\right)$	normal distribution with mean μ and variance σ^2
$N_{\mathscr{X}}(X)$	normal cone of nonempty, closed and convex set $\mathscr{X}$ at $X \in \mathscr{X}$
Ω	probability space of elementary events
$\mathbb{P}$	probability measure
$\mathscr{Q}$	risk envelope
$\mathscr{Q}_{\mathscr{D}}(X)$	set of risk identifiers for X with respect to deviation $\mathscr{D}$
$q_X^+(s)$	upper s-quantile of X
$q_X^-(s)$	lower s-quantile of X
$q_X(s)$	s-quantile interval $\left[q_X^-(s), q_X^+(s)\right]$
$\overline{q}_X(s)$	average s-quantile of X
$\mathscr{R}(X)$	risk measure $\mathscr{R}$ of X
$\mathbb{R}$	set of real numbers
$\mathbb{R}^+$	set of positive real numbers
$\mathbb{R}_0^+$	set of nonnegative real numbers
$\mathbb{R}^n$	set of n-dimensional real vectors
r_0	risk-free rate of return
r_i	rate of return of risky asset $i \in \{1, \dots, n\}$
r_M	rate of return of master fund (market portfolio)
$\mathrm{rc}\mathscr{X}$	recession cone of nonempty, closed and convex set $\mathscr{X}$
$\mathscr{S}(X)$	statistic of X associated with error $\mathscr{E}$
$S(X)$	Shannon entropy of X
$\sigma(X)$	standard deviation of X
$\sigma^2(X)$	variance of X
$\sigma_+(X)$	standard upper semideviation of X
$\sigma_-(X)$	standard lower semideviation of X
σ_M^2	variance of the rate of return of master fund (market portfolio)
sup	essential supremum of X (ess sup X)
$\widehat{\theta}$	statistical estimator of parameter θ
$U(0, 1)$	uniform probability distribution on [0, 1]
$\mathrm{VaR}_\alpha(X)$	value-at-risk (VaR) of X
$\mathrm{VaR}_\alpha^\Delta(X)$	value-at-risk (VaR) deviation of X
X, Y, Z	random variables
$\mathscr{X}$	feasible set of X