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Aims and Scope

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Nonconvex Optimal Control and Variational Problems

Springer

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Preface

This monograph is devoted to the study of nonconvex optimal control and variational problems. It contains a number of recent results obtained by the author in the last 15 years. The Tonelli classical existence theorem in the calculus of variations [81] is based on two fundamental hypotheses concerning the behavior of the integrand as a function of the last argument (derivative): one that the integrand should grow superlinearly at infinity and the other that it should be convex (or exhibit a more special convexity property in case of a multiple integral with vector-valued functions) with respect to the last variable. Moreover, certain convexity assumptions are also necessary for properties of lower semicontinuity of integral functionals which are crucial in most of the existence proofs, although there are some interesting theorems without convexity (see Chap. 16 of [21] and [19, 20, 28, 61, 63]). Since in this book we do not use convexity assumptions on integrands the situation becomes more difficult. We overcome this difficulty using the so-called generic approach which is applied fruitfully in many areas of analysis (see, for example, [6, 67, 69, 71, 72, 99, 106] and the references mentioned there).

According to the generic approach we say that a property holds for a generic (typical) element of a complete metric space (or the property holds generically) if the set of all elements of the metric space possessing this property contains a G_δ everywhere dense subset of the metric space which is a countable intersection of open everywhere dense sets.

In [86, 88] it was shown that the convexity condition is not needed generically and not only for the existence but also for the uniqueness of a solution and even for well-posedness of the problem (with respect to some natural topology in the space of integrands). More precisely, in [86, 88] we considered a class of optimal control problems (with the same system of differential equations, the same functional constraints, and the same boundary conditions) which is identified with the corresponding complete metric space of cost functions (integrands), say F. We did not impose any convexity assumptions. These integrands are only assumed to satisfy the Cesari growth condition. The main result in [86, 88] establishes the existence of an everywhere dense G_δ-set $F' \subset F$ such that for each integrand in F' the corresponding optimal control problem has a unique solution. It should be mentioned that the author obtained this generic existence result in [86] for general Bolza and Lagrange optimal control problems. This result was published in [88].

The next step was done in a joint paper by Alexander Ioffe and the author (see [42]). There we introduced a general variational principle having its prototype in the variational principle of Deville, Godefroy, and Zizler [30]. A generic existence result in the calculus of variations without convexity assumptions was then obtained as a realization of this variational principle. It was also shown in [42] that some other generic well-posedness results in optimization theory known in the literature and their modifications are obtained as a realization of this variational principle.

The work [86, 88] became a starting point of the author's research on optimal control and variational problems without convexity assumptions which have been continued in the last 15 years. Many results obtained during this period of time are presented in the book. Among them generic existence results for different classes of optimal control problems are collected in Chaps. 2–5. Any of these classes of problems is identified with a functional space equipped with a natural complete metric and it is shown that there exists a G_δ everywhere dense subset of the functional space such that for any element of this subset the corresponding optimal control problem possesses a unique solution and that any minimizing sequence converges to this unique solution. These results are obtained as realizations of variational principles which are generalizations or concretization of the variational principle established in [42].

In Chaps. 6–9 we show nonoccurrence of the Lavrentiev phenomenon for many optimal control and variational problems without convexity assumptions.

We say that the Lavrentiev phenomenon occurs for an optimal control problem if its infimum on the full admissible class of trajectory-control pairs is less than its infimum on a subclass of trajectory-control pairs with bounded controls.

The Lavrentiev phenomenon in the calculus of variations was discovered in 1926 by M. Lavrentiev in [45]. There it was shown that it is possible for the variational integral of a two-point Lagrange problem, which is sequentially weakly lower semicontinuous on the admissible class of absolutely continuous functions, to possess an infimum on the dense subclass of C^1 admissible functions that is strictly greater than its minimum value on the admissible class. Since this seminal work, the Lavrentiev phenomenon is of great interest in the calculus of variations and optimal control [1, 8, 9, 21, 25, 26, 35, 49, 53, 60, 78–80]. Nonoccurrence of the Lavrentiev phenomenon was studied in [1, 25, 26, 35, 49, 79, 80]. It should be mentioned that Clarke and Vinter [25] showed that the Lavrentiev phenomenon cannot occur when a variational integrand $f(t, x, u)$ is independent of t.

In Chaps. 6–9 we consider large classes of optimal control problems identified with the corresponding complete metric spaces of integrands $f(t, x, u)$ depending on t. We establish that for most integrands (in the sense of Baire category) the infimum on the full admissible class of trajectory-control pairs is equal to the infimum on a subclass of trajectory-control pairs whose controls are bounded by a certain constant.

In Chaps. 10–12 we study turnpike properties of approximate solutions of variational and optimal control problems. To have this property means, roughly speaking, that the approximate solutions are determined mainly by the integrand (objective function) and are essentially independent of the choice of interval and end point conditions, except in regions close to the end points.

Turnpike properties are well known in mathematical economics. The term was first coined by Samuelson in 1948 (see [77]) where he showed that an efficient expanding economy would spend most of the time in the vicinity of a balanced equilibrium path (also called a von Neumann path).

We study the turnpike property of approximate solutions of the variational problems with integrands f which belong to a complete metric space of functions $\mathcal{M}$. We do not impose any convexity assumption on f. This class of variational problems was studied in Chap. 2 of [99] for integrands $f \in \mathcal{M}_{co}$, where the space $\mathcal{M}_{co}$ consists of all integrands $f \in \mathcal{M}$ which are convex with respect to the last variable (derivative). In Chap. 2 of [99] we showed that the turnpike property holds for a typical integrand $f \in \mathcal{M}_{co}$. In this book we extend the turnpike result of [99] established for the space $\mathcal{M}_{co}$ to the space of integrands $\mathcal{M}$. We also study turnpike properties for a class of discrete-time optimal control problems.

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