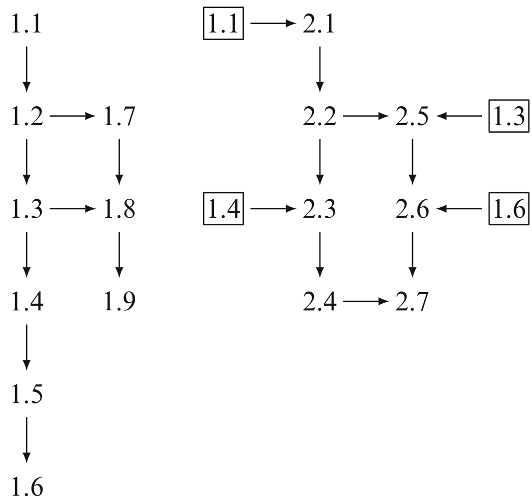
Part I
Calculus and Modeling

The first two chapters contain core material in mathematics: calculus and the basic elements of mathematical modeling. Section 1.1 is an essential preparatory section for the entire book. A significant part of the power of mathematics lies in its capacity for generalization. A single symbol can represent a range of numerical values, allowing the mathematical work to be done on a whole class of problems rather than an individual example. One cannot read any quantitative work in biology or any other science without an ability to understand how symbols are used in a particular context. This is a topic that most readers will find difficult, but one that is essential to master.

Aside from the opening section, Chapter 1 contains five sections on differential calculus and three on integral calculus. Each of these topics includes material on concepts, techniques, and applications. Contrary to the view of most students, calculus is largely a conceptual topic. Concepts are needed to understand the important applications of calculus; hence, the reader should spend enough time on the conceptual material for a thorough understanding. In particular, the derivative concept is essential to understanding dynamical systems, while the definite integral concept is essential for continuous probability distributions. The amount of effort to be expended on techniques is a matter of taste. Differentiation techniques are needed in the rest of the book, but all such cases are fairly elementary. Integration techniques are not needed for the rest of the book. Both of these can be done using computer algebra systems if desired. The applications that appear in Chapter 1 can generally be considered as ends in themselves, and can be accorded as much or as little interest as the reader desires. There are a few exceptions. Related rates (in Section 1.6) are vital background for nondimensionalization, which appears in Section 2.6, and linear approximation (in Section 1.4) is vital background for the nonlinear dynamics that appears in Chapter 5 and 7 and Appendix A.1.

In the preface, I described mathematical modeling as the tendons that connect the muscle of mathematics to the bones of science. Colleagues who teach science and engineering courses often say that their students do not seem to be able to do the mathematics necessary for their subject. The real problem is not so much an inability to do mathematics but an inability to harness the power of mathematics in a scientific context. More or different mathematics will not address this problem; it requires attention to mathematical modeling, which is largely absent from courses in either mathematics or science. This is the purpose of Chapter 2. The first two sections provide basic terminology and ideas. There does not seem to be a lot of material in these sections, and there are very few associated problems. The reader should plan to reread these sections several times while working through the rest of the book, as the ideas in them are hard to understand well without prior experience in mathematical modeling. The remainder of the chapter is divided into sections on mechanistic modeling, which starts with assumptions about the scientific setting, and empirical modeling, which starts with examination of data. The three sections on empirical modeling are of value to anyone who collects and analyzes data, but they are not essential background for the rest of the book. The basic ideas of mechanistic modeling (Section 2.5) are helpful to try to work through, but the reader does not need to be expert on this subject. Most biologists need to be able to read and understand mechanistic models but do not need to be able to create them. The reader of Part III of this book is often asked to interpret mathematical models mechanistically, but is never asked to construct one. In contrast, nondimensionalization (Section 2.6) is a vital skill for anyone who wants to do any work with dynamical systems. It would have been impossible to write a useful introduction to dynamical systems using complete versions of well-known models without relying on the power of nondimensionalization to simplify model analysis.

The accompanying sketch shows the interdependencies of the sections in Part I. Sections 1.1–1.3 and 2.2 are necessary background for the remainder of the book. Sections 1.7 and 1.8 are needed for Part II, while Section 2.5 and 2.6 are needed for Part III. Sections 1.4–1.6, and 1.9 are important topics in calculus, but are not necessary for the rest of the book; similarly, the empirical modeling topics in Section 2.3, 2.4, and 2.7 are not needed later.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