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# Malliavin Calculus and Stochastic Analysis

A Festschrift in Honor of David Nualart

 Springer

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# Preface

David Nualart was born in Barcelona on March 21, 1951. After high school he studied mathematics at the University of Barcelona, from which he obtained an undergraduate degree in 1972 and a PhD in 1975. He was a full professor at the University of Barcelona from 1984 to 2005. He moved to the University of Kansas in 2005, as a Professor in the Department of Mathematics, and was appointed Black-Babcock Distinguished Professor there in 2012.

David Nualart is among the world's most prolific authors in probability theory, with more than 200 research papers, many of which are considered pathbreaking, and several influential monographs and lecture notes. His most famous book is undoubtedly *Malliavin Calculus and Related Topics* (cited more than 530 times on MathSciNet), which has been serving as an ultimate reference on the topic since its publication. Its most recent edition contains two chapters which have become standard references in their own right, on state-of-the-art applications of the Malliavin calculus to quantitative finance and to fractional Brownian motion.

David Nualart has long influenced the general theory of stochastic analysis, including martingale theory, stochastic calculus of variations, stochastic equations, limit theorems, and mathematical finance. In the first part of his scientific life, he contributed to the development of a stochastic calculus for two-parameter martingales, setting the basis of stochastic integration in this context. Subsequently, one of his major achievements in probability theory has been his ability to develop and apply Malliavin calculus techniques to a wide range of concrete, interesting, and intricate situations. For instance, he is at the inception and is recognized as the leader in anticipating stochastic calculus, a genuine extension of the classical Itô calculus to non-adapted integrands. His other contributions to stochastic analysis include results related to integration-by-parts formulas, divergence and pathwise integrals, regularity of the laws of random variables through Malliavin calculus, and the study of various types of stochastic (partial) differential equations.

In the last decades, his research focused largely on the stochastic calculus with respect to Gaussian processes, especially fractional Brownian motion, to which he has become the main contributor. David Nualart's most recent work also includes important results on limit theorems in terms of Malliavin calculus.

David Nualart's prominent role in the stochastic analysis community and the larger mathematics profession is obvious by many other metrics, including membership in the Royal Academy of Exact Physical and Natural Sciences of Madrid since 2003, an invited lecture at the 2006 International Congress of Mathematicians, continuous and vigorous service as editor or associate editor for all the main journals in probability theory, and above all, the great number of Ph.D. students, postdoctoral scholars, and collaborators he has trained and worked with around the world. By being an open-minded, kind, generous, and enthusiastic colleague, mentor, and person, he has fostered a good atmosphere in stochastic analysis. All those working in this area have cause to be grateful and to celebrate the career of David Nualart.

In this context, the book you hold in your hands presents 25 research articles on various topics in stochastic analysis and Malliavin calculus in which David Nualart's influence is evident, as a tribute to his lasting impact in these fields of mathematics. Each article went through a rigorous peer-review process, led by this volume's four editors Jin Feng (Kansas), Yaozhong Hu (Kansas), Eulàlia Nualart (Pompeu Fabra, Barcelona), and Frederi Viens (Purdue) and six associate members of this volume's Editorial Board, Laure Coutin (Toulouse), Ivan Nourdin (Nancy), Giovanni Peccati (Luxembourg), Lluís Quer-Sardanyons (Autònoma, Barcelona), Samy Tindel (Nancy), and Ciprian Tudor (Lille), with the invaluable assistance of many anonymous referees.

The articles' authors represent some of the top researchers in these fields, all of whom are recognized internationally for their contributions to date; many of them were also able to participate in a conference in honor of David Nualart held at the University of Kansas on March 19–21, 2011, on Malliavin calculus and stochastic analysis, with major support from the US National Science Foundation, with additional support from the Department of Mathematics and the College of Liberal Arts and Sciences at the University of Kansas, the Department of Mathematics and the Department of Statistics at Purdue University, and the French National Agency for Research.

As the title of this volume indicates and the topics of many of the articles within emphasize, this Festschrift also serves as a tribute to the memory of Paul Malliavin and his extraordinary influence on probability and stochastic analysis, through the inception and subsequent constant development of the stochastic calculus of variations, known today as the Malliavin calculus. Professor Malliavin passed away in June 2010. He is dearly missed by many as a mathematician, colleague, mentor, and friend. Dan Stroock initially coined the term “Malliavin calculus” around 1980 to describe the stochastic calculus of variations developed by Paul Malliavin, which employs the Malliavin derivative operator. The term has been broadened to describe any mathematical activity using this derivative and related operators on standard or abstract Wiener space as well as, to some extent, calculus based on Wiener chaos expansions. We consider the Malliavin calculus in this broadest sense.

The term “stochastic analysis” originated in its use as the title of the 1978 conference volume edited by Avner Friedman and Mark Pinsky. It described results

on finite- and infinite-dimensional stochastic processes that employ probabilistic tools as well as tools from classical and functional analysis. We understand stochastic analysis as being broadly rooted and applied this way in probability theory and stochastic processes, rather than a term to describe solely analysis results with a probabilistic flavor or origin.

The topics in this volume are divided by theme into five parts, presented from the more theoretical to the more applied. While these divisions are not fundamental in nature and can be interpreted loosely, they crystallize some of the most active areas in stochastic analysis today and should be helpful for readers to grasp the motivations of some of the top researchers in the field.

- Part I covers Malliavin calculus and Wiener space theory, with topics which advance the basic understanding of these tools and structures; these topics are then used as tools throughout the rest of the volume.
- Part II develops the analysis of stochastic differential systems.
- Part III furthers this development by focusing on stochastic partial differential equations and some of their fine properties.
- Part IV also deals largely with stochastic equations and now puts the emphasis on noise terms with long-range dependence, particularly using fractional Brownian motion as a building block.
- Part V closes the volume with articles whose motivations are solving specific applied problems using tools of Malliavin calculus and stochastic analysis.

A number of stochastic analysis methods cut across all of the five parts listed above. Some of these tools include:

- Analysis on Wiener space
- Regularity and estimation of probability laws
- Malliavin calculus in connection to Stein's method
- Variations and limit theorems
- Statistical estimators
- Financial mathematics

As the readers will find out by perusing this volume, stochastic analysis can be interpreted within several distinct fields of mathematics and has found many applications, some reaching far beyond the core mathematical discipline. Many researchers working in probability, often using tools of functional analysis, are still heavily involved in discovering and developing new ways of using the Malliavin calculus, making it one of the most active areas of stochastic analysis today and for some time to come. We hope this Festschrift will serve to encourage researchers to consider the Malliavin calculus and stochastic analysis as sources of new techniques that can advance their research.

The four editors of this Festschrift are indebted to the members of the Editorial Board, Laure Coutin, Ivan Nourdin, Giovanni Peccati, Lluís Quer-Sardanyons, Samy Tindel, and Ciprian Tudor, for their tireless work in selecting and editing

the articles herein, to the many anonymous referees for volunteering their time to discern and help enforce the highest quality standards, and above all to David Nualart, for inspiring all of us to develop our work in stochastic analysis and the Malliavin calculus.

Thank you, David.

Lawrence, Kansas, USA  
Barcelona, Spain  
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Jin Feng and Yaozhong Hu  
Eulalia Nualart  
Frederi Viens



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