

Part 4. Higher Reciprocity Laws

In this part we use the construction of the moduli scheme of elliptic modules, the trace formula, and various forms of the fixed-point formula to obtain various applications. In Chap. 10 we obtain the purity theorem, or Ramanujan conjecture for the cuspidal representations of $\mathrm{GL}(r)$ over a function field with a cuspidal component. In Chap. 11 we show that for the cuspidal automorphic representation π with cuspidal component π_∞ there exists a corresponding Galois representation σ . In Chap. 12 we establish the inverse correspondence, thus that the reciprocity law, relating such automorphic representations with irreducible r -dimensional representations of the Weil group as well as a local analogue, is a bijection. To establish the global converse to the existence theorem $\pi \mapsto \sigma$, we develop in Chap. 13 a simple form of the converse theorem, for automorphic representations with a cuspidal component.