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Yuval Z. Flicker

# Drinfeld Moduli Schemes and Automorphic Forms

The Theory of Elliptic Modules  
with Applications

 Springer

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