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Javad Mashreghi

Derivatives of Inner Functions

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Ouvrez une école, vous fermerez une prison

Victor Hugo

It was in 1934 that Pahlavi High School was established in my hometown, Kashan. In 1946, the school moved to a new building constructed over a rather vast area and featuring an awesome architectural design. The school was renamed Imam Khomeini High School after the 1979 revolution. Over the years, numerous bright minds were trained in the stimulating environment of this school. Before long, the impressive school building had become a reminder of all the great intellectuals who had either studied or taught there. To the utter regret of the latter, however, the building was completely demolished in 1995, only to give way to the current, incomplete one. I dedicate this monograph to all the caring and respected men, teachers and employees alike, who kept the flame of education alight for many years in this institute.

مدرسه‌ای باز کنید، زندانی را خواهید بست.

ویکتور هوگو

دبیرستان پهلوی در شهر من کاشان به سال ۱۳۱۴ تاسیس و در سال ۱۳۲۶ در مکانی نسبتاً وسیع و با معماری بسیار زیبا و خاطره انگیز بنا شد. بعد از انقلاب ۱۳۵۷، نام اولیه دبیرستان به امام خمینی تغییر یافت. این مجموعه در طول سالیان متمادی نخبگان زیادی را در دامن پر مهر خود پرورش داد و بنای با شکوهش یاد و خاطره‌ی مردان بزرگ و گران‌قدری را که در آن تدریس و یا تحصیل نموده بودند زنده نگاه می‌داشت. با کمال تأسف در سال ۱۳۷۴، این بنا به طور کامل تخریب گردید و بنای جدید و ناتمام فعلی به جای آن ساخته شد. این کتاب را به دبیران دلسوز و کارمندان شریفی که مشعل آموزش را در این مرکز برافروخته داشته و به دوش کشیدند تقدیم می‌دارم.

Painting by Mrs. Najmeh Firoozpour

Preface

Infinite Blaschke products were introduced by W. Blaschke in 1915 [9]. In 1929, R. Nevanlinna introduced the class of bounded analytic functions with almost everywhere unimodular boundary values [35]. However, the term *inner function* was coined much later by A. Beurling in his seminal work on the invariant subspaces of the shift operator [8]. The first extensive studies of the properties of inner functions were made by O. Frostman [22], W. Seidel [43] and F. Riesz [40]. Their efforts to understand the zeros and boundary behavior of bounded analytic functions led to the celebrated canonical factorization theorem. The special factorization that we need says that each inner function is the product of a *Blaschke product* and a zero free inner function, the so called *singular inner function*, which is generated by a singular measure residing on the unit circle. Roughly speaking, we can say that the Blaschke product is formed with the zeros of an inner function inside the open unit disc, and the singular part stems from its zeros on the boundary.

In July 2011, E. Fricain and I organized a conference on *Blaschke products and their applications* in the Fields Institute (Toronto). There were several interesting talks about the boundary behavior of inner functions, in particular Blaschke products, and their derivatives. I felt the need to gather some classical results in a short monograph for graduate students and as a handy reference for experts. However, the literature is very vast and it is a difficult task to choose among various important results. For example, the book of P. Colwell [16] can provide a panoramic picture of this subject. Hence, I restricted myself just to the integral means of the derivatives and, even for this narrow subject, I was very selective.

The Fields Institute exclusively supported our conference on Blaschke products, and its direction constantly helped us for the production of the proceedings and this monograph. In particular, I owe profound thanks to Carl Riehm, the Managing Editor of Publications, for his care, guidance, and enthusiastic support. Last but not the least, I would like to deeply thank

Joseph Cima (University of North Carolina), Ian Graham (University of Toronto), and Armen Edigarian (Jagiellonian University) who kindly read the manuscript and made many valuable suggestions. Their remarks enormously improved the quality of text.

Montreal, QC Javad Mashreghi

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