

Fields Institute Communications

VOLUME 65

The Fields Institute for Research in Mathematical Sciences

Fields Institute Editorial Board:

Carl R. Riehm, *Managing Editor*

Edward Bierstone, *Director of the Institute*

Matheus Grasselli, *Deputy Director of the Institute*

James G. Arthur, *University of Toronto*

Kenneth R. Davidson, *University of Waterloo*

Lisa Jeffrey, *University of Toronto*

Barbara Lee Keyfitz, *Ohio State University*

Thomas S. Salisbury, *York University*

Noriko Yui, *Queen's University*

The Fields Institute is a centre for research in the mathematical sciences, located in Toronto, Canada. The Institute's mission is to advance global mathematical activity in the areas of research, education and innovation. The Fields Institute is supported by the Ontario Ministry of Training, Colleges and Universities, the Natural Sciences and Engineering Research Council of Canada, and seven Principal Sponsoring Universities in Ontario (Carleton, McMaster, Ottawa, Toronto, Waterloo, Western and York), as well as by a growing list of Affiliate Universities in Canada, the U.S. and Europe, and several commercial and industrial partners.

For further volumes:

www.springer.com/series/10503

Javad Mashreghi • Emmanuel Fricain
Editors

Blaschke Products and Their Applications



The Fields Institute for Research
in the Mathematical Sciences

 Springer

Editors

Javad Mashreghi
Département de Mathématiques
et de Statistique
Université Laval
Québec, QC
Canada

Emmanuel Fricain
Laboratoire Paul Painlevé
Université des Sciences et Technologies
Lille 1
Villeneuve d'Ascq cedex
France

ISSN 1069-5265

Fields Institute Communications

ISBN 978-1-4614-5340-6

DOI 10.1007/978-1-4614-5341-3

Springer New York Heidelberg Dordrecht London

ISSN 2194-1564 (electronic)

ISBN 978-1-4614-5341-3 (eBook)

Library of Congress Control Number: 2012950139

Mathematics Subject Classification: 30D50, 30D40, 30D55, 30E20, 30E25, 32A36

© Springer Science+Business Media New York 2013

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)



Conference on *Blaschke Products and Their Applications*
Fields Institute
Toronto
July 25 to 29, 2011

Preface

Infinite Blaschke products were introduced by Blaschke in 1915. However, finite Blaschke products, as a subclass of rational functions, has existed long before without being specifically addressed as finite Blaschke products. In 1929, R. Nevanlinna introduced the class of bounded analytic functions with almost everywhere unimodular boundary values. Then the term inner function was coined much later by A. Beurling in his seminal study of the invariant subspaces of the shift operator. The first extensive study of the properties of inner functions was made by W. Blaschke, W. Seidel and O. Frostman. The Riesz technique in extracting the zeros of a function in a Hardy space is considered as the first step of the full canonical factorization of such elements. The disposition of zeros of an inner function is intimately connected with the existence of radial limits of the inner function and its derivatives.

For almost a century, Blaschke products have been studied and exploited by mathematicians. Their boundary behaviour, the asymptotic growth of various integral means of Blaschke products and their derivatives, their applications in several branches of mathematics in particular as solutions to extremal problems, their membership in different function spaces and their dynamics are examples from a long list of active research domains in which they show their face.

With the exclusive help of Fields Institute, we held a conference on Blaschke Products and their Application from July 25 to 29, 2011, at the University of Toronto. The purpose of the conference was to bring together a wide spectrum of mathematicians in this area. With more than 50 specialists and young researchers from around the globe, we had 36 talks. There were 28 one-hour talks and 8 thirty-minute talks. Besides discussing Blaschke products, or more generally inner functions, and their properties, their applications in other domains were also extensively discussed. In particular, the following topics were of primary attention:

- i. Approximation theory (L. Baratchart, A. Boivin, P. Gorkin, V. Prokhorov),
- ii. Boundary values (W. Ross),
- iii. Conformal metrics (O. Roth),
- iv. Critical points (S. Favorov, D. Kraus),
- v. Differential equations (J. Benbourenane, J. Heittokangas),
- vi. Dynamical systems (O. Ivrii),

- vii. Geometry (U. Daepf),
- viii. Harmonic analysis (M. Pap),
 - ix. Hyperbolic geometry (L. Baribeau),
 - x. Integral means (D. Vukotic),
 - xi. Inner functions (A. Nicolau),
 - xii. Interpolation (P. Gorkin, G. Semmler),
- xiii. Morse theory (L. Baratchart),
- xiv. Operator theory (H. Bommier, S. Charpentier, D. Drissi),
 - xv. Pluripotential theory (A. Edigarian, W. Zwonek),
 - xvi. Riemann-Hilbert problem (C. Glader),
- xvii. Ritt's theory (P. Tuen-Wai Ng),
- xviii. Spectral theory of Toeplitz operators (E. Shargorodsky),
 - xix. Theory of analytic functions (I. Chyzykov, R. Fournier, Q. Rahman),
 - xx. Theory of computation (T. McNicholl),
 - xxi. Truncated Toeplitz operators (J. Cima, W. Ross).

These talks were highly appreciated by the participants. It also confirms the fact that Blaschke products impressively appear in a large number of various fields and this conference allowed us to bring together a wide spectrum of prominent mathematicians of different domains.

This proceedings is the outcome of the conference. It contains 15 research-survey papers which are presented in alphabetical order of their titles. We would like to thank all the participants, the authors for their valuable contributions, and the Fields Institute for its unique and generous support of this event.

Javad Mashreghi
Emmanuel Fricain

Contents

Applications of Blaschke Products to the Spectral Theory of Toeplitz Operators	1
Sergei Grudsky and Eugene Shargorodsky	
Approximating the Riemann Zeta-Function by Strongly Recurrent Functions	31
P.M. Gauthier	
A Survey on Blaschke-Oscillatory Differential Equations, with Updates .	43
Janne Heittokangas	
Bi-orthogonal Expansions in the Space $L^2(0, \infty)$	99
André Boivin and Changzhong Zhu	
Blaschke Products as Solutions of a Functional Equation	113
Javad Mashreghi	
Cauchy Transforms and Univalent Functions	119
Joseph A. Cima and John A. Pfaltzgraff	
Critical Points, the Gauss Curvature Equation and Blaschke Products . .	133
Daniela Kraus and Oliver Roth	
Growth, Zero Distribution and Factorization of Analytic Functions of Moderate Growth in the Unit Disc	159
Igor Chyzykhov and Severyn Skaskiv	
Hardy Means of a Finite Blaschke Product and Its Derivative	175
Alan Gluchoff and Frederick Hartmann	
Hyperbolic Derivatives Determine a Function Uniquely	187
Line Baribeau	
Hyperbolic Wavelets and Multiresolution in the Hardy Space of the Upper Half Plane	193
Hans G. Feichtinger and Margit Pap	

Norms of Composition Operators Induced by Finite Blaschke Products on Möbius Invariant Spaces	209
María J. Martín and Dragan Vukotić	
On the Computable Theory of Bounded Analytic Functions	223
Timothy H. McNicholl	
Polynomials Versus Finite Blaschke Products	249
Tuen Wai Ng and Chiu Yin Tsang	
Recent Progress on Truncated Toeplitz Operators	275
Stephan Ramon Garcia and William T. Ross	