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Inverse Acoustic and Electromagnetic Scattering Theory

Third Edition

Springer

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From Rainer for Marcus

As far as my eyes can see
There are shadows approaching me
And to those I left behind
I wanted you to know
You've always shared my deepest thoughts
I'll miss you when I go
And oh when I'm old and wise
Bitter words mean little to me
Like autumn winds will blow right through me
And someday in the mist of time
When they asked me if I knew you
I'd smile and say you were a friend of mine
And the sadness would be lifted from my eyes

Alan Parsons

Preface to the Third Edition

Since the second edition of our book appeared fourteen years ago, the field of inverse scattering theory has continued to be an active and growing area of applied mathematics. In this third edition we have tried to bring our book up to date by including many of the new developments in the field that have taken place during this period. We again have made no effort to cover all of the many new directions in inverse scattering theory but rather have restricted ourselves to a selection of those developments that we have either participated in or are a natural development of material discussed in previous editions. We have also continued to emphasize simplicity over generality, e.g. smooth domains instead of domains with corners, isotropic media rather than anisotropic media, standard boundary conditions rather than more generalized ones, etc. By so doing, we hope that our book will continue to serve as a basic introduction to the field of inverse scattering theory.

In order to bring our book up to date, considerable changes have been made to the second edition. In particular, new sections have been added on the linear sampling and factorization methods for solving the inverse scattering problem as well as expanded treatments of iteration methods and uniqueness theorems for the inverse obstacle problem.These additions have also required us to expand our presentation on both transmission eigenvalues and boundary integral equations in Sobolev spaces. These changes in turn suggest a more integrated view of inverse scattering theory. In particular, what was previously referred to as the Colton–Monk and Kirsch–Kress methods, respectively, are now viewed as two examples of what are called decomposition methods. From this point of view the techniques of iteration, decomposition and sampling form a natural trilogy of methods for solving inverse scattering problems. Although a few results from the second edition have been removed due to the fact that we now consider them to be obsolete, for historical reasons we have tended to do so sparingly.

We hope that this new edition of our book will continue to serve readers who are already in the field of inverse scattering theory as well as to attract newcomers to this beautiful area of applied mathematics.

Newark, Delaware — David Colton
Göttingen, Germany — Rainer Kress

Preface to the Second Edition

In the five years since the first edition of this book appeared, the field of inverse scattering theory has continued to grow and flourish. Hence, when the opportunity for a second edition presented itself, we were pleased to have the possibility of updating our monograph to take into account recent developments in the area. As in the first edition, we have been motivated by our own view of inverse scattering and have not attempted to include all of the many new directions in the field. However, we feel that this new edition represents a state of the art overview of the basic elements of the mathematical theory of acoustic and electromagnetic inverse scattering.

In addition to making minor corrections and additional comments in the text and updating the references, we have added new sections on Newton's method for solving the inverse obstacle problem (Section 5.3), the spectral theory of the far field operator (Section 8.4), a proof of the uniqueness of the solution to the inverse medium problem for acoustic waves (Section 10.2) and a method for determining the support of an inhomogeneous medium from far field data by solving a linear integral equation of the first kind (Section 10.7).

We hope that this second edition will attract new readers to the beautiful and intriguing field of inverse scattering.

Newark, Delaware — David Colton
Göttingen, Germany — Rainer Kress

Preface to the First Edition

It has now been almost ten years since our first book on scattering theory appeared [64]. At that time we claimed that "in recent years the development of integral equation methods for the direct scattering problem seems to be nearing completion, whereas the use of such an approach to study the inverse scattering problem has progressed to an extent that a 'state of the art' survey appears highly desirable". Since we wrote these words, the inverse scattering problem for acoustic and electromagnetic waves has grown from being a few theoretical considerations with limited numerical implementations to a well developed mathematical theory with tested numerical algorithms. This maturing of the field of inverse scattering theory has been based on the realization that such problems are in general not only nonlinear but also improperly posed in the sense that the solution does not depend continuously on the measured data. This was emphasized in [64] and treated with the ideas and tools available at that time. Now, almost ten years later, these initial ideas have developed to the extent that a monograph summarizing the mathematical basis of the field seems appropriate. This book is our attempt to write such a monograph.

The inverse scattering problem for acoustic and electromagnetic waves can broadly be divided into two classes, the inverse obstacle problem and the inverse medium problem. In the inverse obstacle problem, the scattering object is a homogeneous obstacle with given boundary data and the inverse problem is to determine the obstacle from a knowledge of the scattered field at infinity, i.e., the far field pattern. The inverse medium problem, in its simplest form, is the situation when the scattering object is an inhomogeneous medium such that the constitutive parameters vary in a continuous manner and the inverse problem is to determine one or more of these parameters from the far field pattern. Only the inverse obstacle problem was considered in [64]. In this book we shall consider both the inverse obstacle and the inverse medium problem using two different methods. In the first method one looks for an obstacle or parameters whose far field pattern best fits the measured data whereas in the second method one looks for an obstacle or parameters whose far field pattern has the same weighted averages as the measured data. The theoretical and numerical development of these two methods for solving the inverse scattering problem for acoustic and electromagnetic waves is the basic subject matter of this book.

We make no claim to cover all the many topics in inverse scattering theory for acoustic and electromagnetic waves. Indeed, with the rapid growth of the field, such a task would be almost impossible in a single volume. In particular, we have emphasized the nonlinear and improperly posed nature of the inverse scattering problem and have paid only passing attention to the various linear methods which are applicable in certain cases. This view of inverse scattering theory has been arrived at through our work in collaboration with a number of mathematicians over the past ten years, in particular Thomas Angell, Peter Hähner, Andreas Kirsch, Ralph Kleinman, Peter Monk, Lassi Päivärinta, Lutz Wienert and Axel Zinn.

As with any book on mathematics, a basic question to answer is where to begin, i.e., what degree of mathematical sophistication is expected of the reader? Since the inverse scattering problem begins with the asymptotic behavior of the solution to the direct scattering problem, it seems reasonable to start with a discussion of the existence and uniqueness of a solution to the direct problem. We have done this for both the Helmholtz and the Maxwell equations. Included in our discussion is a treatment of the numerical solution of the direct problem. In addition to a detailed presentation of the direct scattering problem, we have also included as background material the rudiments of the theory of spherical harmonics, spherical Bessel functions, operator valued analytic functions and ill-posed problems (This last topic has been considerably expanded from the brief discussion given in [64]). As far as more general mathematical background is concerned, we assume that the reader has a basic knowledge of classical and functional analysis.

We have been helped by many people in the course of preparing this book. In particular, we would like to thank Wilhelm Grever, Rainer Hartke and Volker Walther for reading parts of the manuscript and Peter Hähner for his many valuable suggestions for improvements. Thanks also go to Ginger Moore for doing part of the typing. We would like to acknowledge the financial support of the Air Force Office of Scientific Research and the Deutsche Forschungsgemeinschaft, both for the long-term support of our research as well as for the funds made available to us for regular visits between Newark and Göttingen to nurture our collaboration. Finally, we want to give special thanks to our friends and colleagues Andreas Kirsch and Peter Monk. Many of the results of this book represent joint work with these two mathematicians and their insights, criticism and support have been an indispensable component of our research efforts.

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