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Quantum Plasmadynamics

Magnetized Plasmas

 Springer

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Preface

The motivation and background for the two-volume book on Quantum Plasma-dynamics (QPD) were explained in the Preface to volume 1 [1]. In brief, my objective in QPD is to synthesize quantum electrodynamics (QED) and the kinetic theory of plasmas. My interest in doing this has extended over more than four decades, as an ongoing but secondary research topic. I have used the development of QPD as a training tool for some of my students in theoretical physics. As a result, my collaborators have mostly been research students, who have written theses either partially or completely on problems in this field (including Wilson Sy and Ray Stoneham in the 1970s, Andrew Parle, Leith Hayes, Peter Robinson and Michelle Storey (née Allen) in the 1980s, Whayne Padden, Qinghuan Luo, Stephen Hardy and Malcolm Kennett in the 1990s, and Jock McOrist, Alex Judge and Matthew Verdon in the 2000s). Occasional international visitors (John Kirk, Jan Kuijpers, V.V. Zheleznyakov) were collaborators briefly in earlier years. Longer term collaborations have been with Jeanette Weise, since 1995, and with Mushtaq Ahmed for 2 years (2009–2010).

Volume 2 is essentially an extension of the theory in volume 1 from unmagnetized plasmas to magnetized plasma. Both volumes consist of two parts, with the first part concerned with a covariant reformulation of nonquantum plasma kinetic theory, and the second part concerned with the use of QED to calculate plasma processes and plasma responses. The writing of volume 2 has taken me longer than originally anticipated: most of the material in Chaps. 1–8 already existed in some form 4 years ago, when volume 1 was completed. A major part of the delay has been due to my desire to derive a completely general form for the response of a relativistic quantum electron gas for the magnetized case. The aim was to generalize the results in Chap. 9 of volume 1 to the magnetized case, resulting in Chap. 9 of this volume. These new results have been applied to only a few special cases, and there is much more to be explored relating to dispersion in relativistic quantum magnetized plasmas. I had originally intended to include a tenth chapter, analogous to Chap. 10 of volume 1, extending the theory to a magnetized neutrino plasma, based on [2], and to a magnetized boson plasma, based on [3,4], but decided to omit this material.

One specific problem that delayed the completion of this volume concerns the relation between QPD and quantum fluid theory (QFT), on which there has been a rapidly expanding literature over the past decade or so. I have included a short section (§ 1.5) on QFT in this volume. The problem concerns spin-dependent plasmas: I have been unable to identify how the correct relativistic quantum (QPD) result reproduces the quasi-classical QFT result (§ 9.6). There remains a need to justify (or otherwise) extensions of QFT to include spin in a magnetized electron gas.

Sydney, Australia

Don Melrose

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