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Classical Mechanics

Kinematics and Statics

 Springer

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Preface

This book belongs to a series of three books written simultaneously (the remaining two are titled *Classical Mechanics: Dynamics* and *Classical Mechanics: Applied Mechanics and Mechatronics*). This book's *triad* attempts to cover different subjects in classical mechanics and creates a link between them by introducing them from the same root. The classical mechanics approach extended to the study of electro–magneto–mechanical systems is also emphasized. Another important objective in writing the series of three volumes with the repeated title *Classical Mechanics* was to include and unify sometimes different approaches to the subject in the English, Russian, Polish, and German literature. This explains why sometimes the list of references includes works written either in Russian or Polish (the English literature is now easily available using Google or Scopus). Although the list of references includes works written not necessarily in English, the way of the book material presentation does not require to read the reference sources.

This first volume contains problems of classical mechanics including kinematics and statics. It is recommended as a textbook for undergraduate and graduate students in mechanical and civil engineering and applied physics as well as for researchers and engineers dealing with mechanics. It could also be a main reference for other courses, but it is suited for a course in statics and dynamics.

In Chap. 1 the fundamental principles of mechanics are formulated, illustrated, and discussed. In the introduction, the general historical path of the development of mechanics and its pioneers is described with an emphasis on mechanical modeling of planetary motion and Newtonian mechanics. Three of Newton's laws are formulated and discussed. The definitions of force and mass are given, and then a classification of forces is introduced. Next, the principles of mechanics are given and briefly discussed. In addition, the impact of classical mechanics on electrodynamics (Maxwell's equations) and relativistic mechanics (Einstein's theories) is presented. Kinetic units, and in particular the principal SI units, are introduced and discussed. In Sect. 1.2, D'Alembert's principle is introduced, illustrated, and discussed. In Sect. 1.3, the principle of virtual work is derived and illustrated. In Sect. 1.4, the increment and the variation of a function are presented, and two examples supporting the introduced theoretical background are given.

Chapter 2 is devoted to statics. First, a concept of equilibrium is introduced, including the formulation of several theorems. The notations of moment of force about a point and about an axis are illustrated, statically determinate and indeterminate problems are defined, and the “freezing principle” is described. In Sect. 2.2, the geometrical equilibrium conditions of a plane force system are introduced and discussed. The force polygon and funicular polygon are introduced and illustrated through two examples. In Sect. 2.3, the geometrical conditions for a space system are formulated. In particular, Poinso’s method is described. Analytical equilibrium conditions are given in Sect. 2.4. The three-moments theorem is formulated and proved. Two reduction invariants as well as the fields of forces are defined. Two theorems regarding mechanical systems with parallel forces are formulated and proved, among others. In Sect. 2.5, mechanical interactions, constraints, and supports are described, and examples of supports carrying three-dimensional systems of forces are graphically illustrated. In addition, three computational examples are shown. Reductions of a space force system to a system of two skew forces (Sect. 2.6) and to a wrench (Sect. 2.7) are carried out. Theoretical considerations are supported by figures and examples, and two reduction invariants are also introduced. In Sect. 2.8, the phenomenon of friction is described including limiting and fully developed friction forces, three Coulomb laws, sliding friction, and rolling resistance. Three illustrative examples are also given. The paradoxical behavior of bodies coupled by friction is presented and discussed in Sect. 2.9, which includes the following examples: (a) supply of energy by means of friction; (b) Coulomb friction as a force exciting rigid body motion; (c) Coulomb friction as viscous damping. Two different approaches to the Euler formula derivation are shown in Sect. 2.10 while dealing with friction of strings wrapped around a cylinder. Friction models are presented and discussed in Sect. 2.11. First, some friction models used in research and frequently found in the literature are presented, and then particular attention is paid to the so-called CCZ (Coulomb–Contensou–Zhuravlev) three-dimensional friction model. Some computational examples putting emphasis on the coupling of sliding and rotation of a body moving on an inclined plane are included.

Chapter 3 is devoted to the geometry of masses. In Sect. 3.1, basic concepts are introduced including mass center, moment of inertia of a system of particles with respect to a plane (an axis), polar moment of inertia. Four illustrative computational examples are provided. Centroids of common shapes of areas, lines, and volumes are given in tables. Two Pappus–Guldinus rules are formulated and supplementary examples of their applications are provided. In Sect. 3.2, the moments of inertia (second moments) are discussed, and formulas for their values for common geometric figures and three-dimensional bodies are included in tables. The inertia matrix and its transformations are discussed in Sect. 3.3. Steiner theorems are formulated, and an illustrative example is provided. In Sect. 3.4, principal axes and principal moments on a plane are defined, and in particular the plane inertia circle (Mohr’s circle) is discussed. The inertia tensor, principal axes of inertia, and ellipsoid of inertia are the theme of Sect. 3.5. In particular, an ellipsoid of

inertia of a body, principal axes of inertia, invariants of an inertia tensor, and inertia triangle inequalities are illustrated and discussed. Section 3.6 presents the properties of principal and principal centroidal axes of inertia, whereas Sect. 3.7 addresses problems related to the determination of moments of inertia of a rigid body.

The kinematics of a particle, the curvilinear and normal coordinates, and kinematic pairs and chains constitute the focus of Chap. 4. In Sect. 4.1, the motion of a particle and trajectory (path) of motion are defined including its velocity and acceleration. Section 4.2 deals with selected problems of plane motion of a particle putting emphasis on circular, rectilinear and curvilinear motion and the vectorial approach. Section 4.3 focuses on introduction and application of rectangular and curvilinear coordinates in space. It includes the classification of a particle's motion with respect to acceleration. In Sect. 4.4, the concept of natural coordinates is illustrated and discussed. The motion of a radius vector and rectangular and curvilinear coordinates in space are introduced and their properties and applications described. Both vector and tensor notations are used, and in particular covariant and contravariant unit vectors are applied. Orthogonal and orthonormal bases are introduced, and then displacement, velocity, and acceleration components are defined via the kinetic energy of a particle. Unit vectors and their first derivatives are derived for rectangular, cylindrical, and spherical coordinate systems. The position, velocity, and acceleration of a particle in rectangular, cylindrical, spherical, and arbitrary curvilinear coordinates are reported. Two illustrative examples are given. Natural coordinates (velocities and accelerations, the Darboux vector, torsion of a curvature, Frenet–Serret formulas, and examples) are studied in Sect. 4.4. Kinematic pairs and chains, joint variables, and the Denavit–Hartenberg convention are considered in Sect. 4.5. Definitions of a class of kinematic pairs, a mechanism, and a group are introduced, and low-order kinematic pairs are presented in a table. Finally, Sect. 4.6 provides a classification of problems in kinematics.

In Chap. 5, the kinematics of a rigid body and the composite motion of a point (particle) are studied. In Sect. 5.1, translational and rotational motions are considered. The study includes the movement of a rigid body in a three-dimensional space and the definition of degree of freedom. In particular, angular velocities and angular accelerations as vectors and the notation of a vector of small rotation are illustrated. Planar motion is studied in Sect. 5.2. It is demonstrated that planar motion can be treated as a composition (geometric sum) of translational and rotational motions, and the first of Euler's theorem is formulated. Moving and fixed centrode concepts are illustrated and discussed. Two theorems regarding the instantaneous points of zero velocity (point C) and zero acceleration (point S) are formulated and then proved. Various vector methods of velocity and acceleration determination on the basis of a point's planar motion are presented and illustrated analytically and geometrically. Burmester's theorems are formulated and their applications are illustrated. A few illustrative examples are provided. In Sect. 5.3 (5.3), a composite point motion in three-dimensional (two-dimensional) space is illustrated and analyzed. The general motion of a rigid body in three-dimensional

space is studied in Sect. 5.5. Following a brief introduction, the angular velocity and angular acceleration of a rigid body are defined using both vector and tensor calculus. Euler's proposal is introduced rigorously, and Euler's formula is derived. In particular, the Eulerian angles are illustratively introduced and explained step by step. A rotation matrix that is a product of three matrices of elementary rotations is derived, and their elements are explicitly given. The matrix's non-commutivity is discussed, among other topics. An important theorem is formulated and proved. In Sect. 5.5.4, Eulerian angles are introduced, whereas in Sect. 5.5.5, Euler's kinematic equations are formulated, and both Euler's angles and angular velocities are derived and graphically presented. The displacement of a rigid body with one point fixed is studied in a subsequent section. Several basic theorems regarding the displacement and rotation of a rigid body are formulated in Sect. 5.5.7. In the subsequent short subsections, the parallel translation and rotation of a rigid body and a homogenous transformation, kinematic states of a rigid body's velocity and acceleration in translational motion and in motion about a point are illustrated and studied. In particular, rotational, centripetal, and normal accelerations are analyzed. Then, in Sects. 5.5.12 and 5.5.13, the velocities and accelerations in the motion of a body about a fixed axis and in various coordinate systems are studied. The velocities of a point of a rigid body in various coordinate systems are considered in Sect. 5.5.14, the regular precessions of a rigid body are analyzed in Sect. 5.5.15 (two additional examples support the introduced theoretical considerations). Screw motion is illustrated and discussed in Sect. 5.5.16, whereas the geometrical interpretation of velocity and acceleration of a point of a rigid body in general motion is given in Sect. 5.5.17. The latter subsection also includes a few important theorems, sometimes rigorously proved. The composite motion of a rigid body is analyzed in Sect. 5.6. It consists of the composition of two instantaneous translational (rotational) motions, a couple of instantaneous rotations, and the composition of rotational motions of a rigid body about intersecting axes. Two theorems are formulated and proved, among others.

In Chap. 6, the kinematics of a deformable body is studied. In Sect. 6.1, the role of tensor notation in mechanics is described. In Sect. 6.2, particular attention is paid to the stress tensor. Symmetric and asymmetric tensors are introduced, and their actions are explained analytically and geometrically. Elastic body deformation is studied via introduction of a tensor governing the velocity of body deformation. Two simple examples of theoretical considerations are provided. The definition of Clapeyron systems is introduced, and then Betti's and Castigliano's theorems are formulated and proved.

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Łódź and Darmstadt

Jan Awrejcewicz

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