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Regularity Concepts in Nonsmooth Analysis

Theory and Applications

Springer

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To the soul of my dear father, Ahmed, who died in 1994 and who always believed in me and encouraged me to study mathematics.

To my dear mother Fatma, who has always stood behind all my successes.

To my wife Leila, my children Saged, Kawtar, and Yakine, and to my brothers and sisters and all the members of my family.

Preface

The term *nonsmooth analysis theory* had been used in the 1970s by F. Clarke when he studied and applied the differential properties of functions and sets that are not differentiable in the usual sense. Since Clarke's work, the field of nonsmooth analysis theory has known a considerable expansion, namely with the appearence of an important concept which is the concept of "*regularity* " (regularity of functions and regularity of sets). The primary motivation for introducing regularity notions is to obtain equalities in calculus rules involving various constructs in nonsmooth analysis. The first notion of regularity appeared in Clarke's work (in the 1970s) to ensure equality form in the calculus rules of the Clarke subdifferential for Lipschitz continuous functions.

Many investigators (Rockafellar, Mordukhovich, Thibault, Poliquin et al.) have since then introduced and used many other notions of regularity in the development of nonsmooth analysis theory.

In the last decades, regularity concepts played an increasing role in the applications of nonsmooth analysis such as differential inclusions, optimization, variational inequalities, as well as in nonsmooth analysis itself. Consequently, it is becoming more and more desirable to introduce regularity, at an early stage of study, to graduate students and young researchers in order to familiarize them with the basic concepts and their applications. This book is devoted to the study of various regularity notions in nonsmooth analysis and their applications. To the best of my knowledge, the present work is the first thorough study of the regularity of functions, sets, and multifunctions as well as their important applications to differential inclusions and variational inequalities.

This book is divided into three parts. In the first part, we present an accessible and thorough introduction to nonsmooth analysis theory. Main concepts and some useful results are stated and illustrated through examples and exercises.

In Part II, the most important and recent results of various regularity concepts of sets, functions, and set-valued mappings, in nonsmooth analysis theory are presented. These results include some that have been demonstrated in different works that were published either singly (see [39, 44, 45, 48]), or in collaboration with Thibault (see [58–63]).

Part III contains six chapters, each of which addresses a different application of nonsmooth analysis theory. These applications are the fruit of research that I conducted either singly (see [42, 43]) or in collaboration with various researchers in the field (see [53–55, 58, 64]).

Batna, Algeria Messaoud Bounkhel

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