

**Materials Research and Engineering**  
**Edited by B. Ilshner and N. J. Grant**

Ismail C. Noyan   Jerome B. Cohen

# Residual Stress

Measurement by Diffraction  
and Interpretation

With 160 Figures and 31 Tables



Springer Science+  
Business Media, LLC

Dr. ISMAIL CEVDET NOYAN

Thomas J. Watson Research Center, IBM  
P.O. Box 218  
Yorktown Heights, NY 10598/USA

Prof. JEROME B. COHEN

Dept. of Materials Science and Engineering  
Dean, The Technological Institute, Northwestern University  
Evanston, IL 60201/USA

*Series Editors*

Prof. BERNHARD ILSCHNER

Laboratoire de Métallurgie Mécanique  
Département des Matériaux, Ecole Polytechnique Fédérale  
CH-1007 Lausanne/Switzerland

Prof. NICHOLAS J. GRANT

Dept. of Materials Science and Engineering, MIT  
Cambridge, MA 02139/USA

Library of Congress Cataloging in Publication Data.

Noyan, I. C. (Ismail Cevdet), 1956–. Residual stress.

(Materials research and engineering)

Includes bibliographies and index.

1. Residual stresses—Measurement. 2. Nondestructive testing.

I. Cohen, J. B. (Jerome Bernard), 1932–. II. Title. III. Series.

TA417.6.N68 1986 620.1'123'0287 86-21919

ISBN 978-1-4613-9571-3 ISBN 978-1-4613-9570-6 (eBook)

DOI 10.1007/978-1-4613-9570-6

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks.

© Springer Science+Business Media New York 1987

Originally published by Springer-Verlag New York, Inc. in 1987

Softcover reprint of the hardcover 1st edition 1987

The use of registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typesetting: With a system of Springer Produktions-Gesellschaft, Berlin

Dataconversion: Brühlsche Universitätsdruckerei, Giessen

Offsetprinting: Ruksaldruck, Berlin. Bookbinding: Lüderitz & Bauer, Berlin

2161/3020-543210

# Preface

As industries find that the market for their goods and services is often as closely connected to their quality as to their price, they become more interested in inspection and quality control. Non-destructive testing is one aspect of this topic; the subject of this book is a sub-field of this domain. The techniques for measuring residual stresses have a long history for a technological subject. Yet, in the last decade or so there has been renewed and vigorous interest, and, as a result of this, there has been considerable progress in our understanding and in our methods. It seemed a proper time to bring the new material together in an organized form suitable for a course or for self-teaching, hence this book.

After an initial introduction to the qualitative ideas concerning the origin, role, and measurement of residual stresses, we follow with chapters on classical elasticity and the relatively new subject of microplasticity. These are primarily introductory or review in nature, and the reader will find it important to consider further the quoted references if he is to be involved in a continuing basis in this area. There follows a chapter on diffraction theory, and then we fuse these subjects with a chapter on diffraction techniques for measuring stresses and strains which at present is our most general tool for non-destructive evaluation in this area. In Chap. 6 we explore how to evaluate errors in such a measurement, a topic almost as important as the stress itself and certainly vital in automating the measurement. Practical examples are described in Chap. 7 and means for measuring the strain distribution, in Chap. 8, in contrast to the measurement of average values described in the earlier chapters.

The Appendices include solutions to the problems given at the end of some of the chapters, Fourier analysis, and sources of useful data.

We emphasize diffraction because in our opinion it is the most well understood and reliable tool available for the measurement of stresses. Of course our own background in diffraction colors our opinion, but we hope this book will reveal to the reader why we feel this way.

Dr. M. James provided Appendices E and F, and Mr. W. P. Evans supplied many practical examples for Chap. 7. Both gave us helpful comments on the manuscript.

Both authors thank our many co-workers in this field, Drs. R. Marion, M. James, H. Dölle, W. Schlosberg, Prof. Jai Wen Ho, Mrs. Rui Mai Zhong, Messrs. T. Devine, W. Evans, and P. Rudnik. The two of us have enjoyed (most of the time) working together over the past five years on research in this field and on this book. One of us (I. C. N.) especially acknowledges the support of his family.

Evanston, IL, April 1987

I. C. Noyan  
J. B. Cohen

**Dedication**

We dedicate this book to the U.S. Office of Naval Research and particularly to Dr. Bruce MacDonald of this office. His foresight led us into this field and his helpful encouragement and support led us along many interesting avenues of research on residual stresses.

# Contents

<b>1 Introduction</b> . . . . .	1
1.1 The Origin of Stresses . . . . .	1
1.2 Methods of Measuring Residual Stresses . . . . .	4
1.3 Some Examples of Residual Stresses . . . . .	7
References . . . . .	12
<b>2 Fundamental Concepts in Stress Analysis</b> . . . . .	13
2.1 Introduction . . . . .	13
2.2 Definitions . . . . .	13
2.3 Stress and Strain . . . . .	14
2.4 Forces and Stresses . . . . .	15
2.5 Displacements and Strains . . . . .	17
2.6 Transformation of Axes and Tensor Notation . . . . .	20
2.7 Elastic Stress-Strain Relations for Isotropic Materials . . . . .	25
2.8 Structure of Single Crystals . . . . .	28
2.9 Elastic Stress-Strain Relations in Single Crystals . . . . .	32
2.10 Equations of Equilibrium . . . . .	37
2.11 Conditions of Compatibility . . . . .	38
2.12 Basic Definitions in Plastic Deformation . . . . .	39
2.13 Plastic Deformation of Single Crystals . . . . .	41
2.14 Deformation and Yielding in Inhomogeneous Materials . . . . .	44
Problems . . . . .	45
Bibliography . . . . .	46
<b>3 Analysis of Residual Stress Fields Using Linear Elasticity Theory</b> . . . . .	47
3.1 Introduction . . . . .	47
3.2 Macroresidual Stresses . . . . .	47
3.3 Equations of Equilibrium for Macro stresses . . . . .	51
3.4 Micro stresses . . . . .	52
3.5 Equations of Equilibrium for Micro- and Pseudo-Macro stresses . . . . .	54
3.6 Calculation of Micro- and PM Stresses . . . . .	56
3.7 The Total Stress State in Surface Deformed Multiphase Materials . . . . .	61
3.8 Macroscopic Averages of Single Crystal Elastic Constants . . . . .	62
3.9 The Voigt Average . . . . .	63
3.10 The Reuss Average . . . . .	65
3.11 Other Approaches to Elastic Constant Determination . . . . .	66
3.12 Average Diffraction Elastic Constants . . . . .	69

Summary . . . . .	73
References . . . . .	73
<b>4 Fundamental Concepts in X-ray Diffraction . . . . .</b>	<b>75</b>
4.1 Introduction . . . . .	75
4.2 Fundamentals of X-rays . . . . .	75
4.3 Short-wavelength Limit and the Continuous Spectrum . . . . .	76
4.4 Characteristic Radiation Lines . . . . .	77
4.5 X-ray Sources . . . . .	80
4.6 Absorption of X-rays . . . . .	82
4.7 Filtering of X-rays . . . . .	84
4.8 Scattering of X-rays . . . . .	84
4.9 Scattering from Planes of Atoms . . . . .	86
4.10 The Structure Factor of a Unit Cell . . . . .	87
4.11 Experimental Utilization of Bragg's Law . . . . .	89
4.12 Monochromators . . . . .	90
4.13 Collimators and Slits . . . . .	91
4.14 Diffraction Patterns from Single Crystals . . . . .	93
4.15 Diffraction Patterns from Polycrystalline Specimens . . . . .	94
4.16 Basic Diffractometer Geometry . . . . .	95
4.17 Intensity of Diffracted Lines for Polycrystals . . . . .	97
4.18 Multiplicity . . . . .	98
4.19 Lorentz Factor . . . . .	98
4.20 Absorption Factor . . . . .	100
4.21 Temperature Factor . . . . .	103
4.22 X-ray Detectors . . . . .	103
4.23 Deadtime Correction for Detection Systems . . . . .	108
4.24 Total Diffracted Intensity at a Given Angle $2\theta$ . . . . .	109
4.25 Depth of Penetration of X-rays . . . . .	110
4.26 Fundamental Concepts in Neutron Diffraction . . . . .	111
4.27 Scattering and Absorption of Neutrons . . . . .	114
Problems . . . . .	116
Bibliography and References . . . . .	116
<b>5 Determination of Strain and Stress Fields by Diffraction Methods . . . . .</b>	<b>117</b>
5.1 Introduction . . . . .	117
5.2 Fundamental Equations of X-ray Strain Determination . . . . .	117
5.3 Analysis of Regular "d" vs. $\sin^2\psi$ Data . . . . .	119
5.4 Determination of Stresses from Diffraction Data . . . . .	120
5.5 Biaxial Stress Analysis . . . . .	122
5.6 Triaxial Stress Analysis . . . . .	125
5.7 Determination of the Unstressed Lattice Spacing . . . . .	126
5.8 Effect of Homogeneity of the Strain Distribution and Specimen Anisotropy . . . . .	130
5.9 Average Strain Data from Single Crystal Specimens . . . . .	131

5.10 Interpretation of the Average X-ray Strain Data Measured from Polycrystalline Specimens . . . . . 136

5.11 Interpretation of Average Stress States in Polycrystalline Specimens . 137

5.12 Effect of Stress Gradients Normal to the Surface on  $d$  vs.  $\sin^2\psi$  Data . . . . . 140

5.13 Experimental Determination of X-ray Elastic Constants . . . . . 145

5.14 Determination of Stresses from Oscillatory Data . . . . . 153

5.15 Stress Measurements with Neutron Diffraction . . . . . 154

5.16 Effect of Composition Gradients with Depth . . . . . 157

5.17 X-ray Determination of Yielding . . . . . 159

5.18 Summary . . . . . 160

Problem . . . . . 162

References . . . . . 162

**6 Experimental Errors Associated with the X-ray Measurement of Residual Stress . . . . . 164**

6.1 Introduction . . . . . 164

6.2 Selection of the Diffraction Peak for Stress Measurements . . . . . 164

6.3 Peak Location . . . . . 166

    6.3.1 Half-Value Breadth and Centroid Methods . . . . . 167

    6.3.2 Functional Representations of X-ray Peaks . . . . . 168

    6.3.3 Peak Determination by Fitting a Parabola . . . . . 171

    6.3.4 Determination of Peak Shift . . . . . 175

6.4 Determination of Peak Position for Asymmetric Peaks . . . . . 178

6.5 Statistical Errors Associated with the X-ray Measurement of Line Profiles . . . . . 181

6.6 Statistical Errors in Stress . . . . . 186

    6.6.1 The  $\sin^2\psi$  Technique . . . . . 186

    6.6.2 Two-Tilt Technique . . . . . 187

    6.6.3 Triaxial Stress Analysis . . . . . 187

    6.6.4 Statistical Errors in X-ray Elastic Constants . . . . . 189

6.7 Instrumental Errors in Residual Stress Analysis . . . . . 190

    6.7.1 Variation of the Focal Point with  $\theta$  and  $\psi$  . . . . . 191

    6.7.2 Effect of Horizontal Divergence on Focusing . . . . . 192

    6.7.3 Effect of Vertical Beam Divergence . . . . . 195

    6.7.4 Effect of Specimen Displacement . . . . . 196

    6.7.5 Effect of  $\psi$ -axis not Corresponding to the  $2\theta$ -axis . . . . . 199

    6.7.6 Error Equations for the  $\psi$ -Goniometer . . . . . 200

    6.7.7 Effect of Errors in the True Zero Position of the  $\psi$ -axis . . . . . 202

    6.7.8 Alignment Procedures . . . . . 203

6.8 Corrections for Macrostress Gradients . . . . . 205

6.9 Corrections for Layer Removal . . . . . 206

6.10 Summary . . . . . 208

Problems . . . . . 209

References . . . . . 209



<b>7 The Practical Use of X-ray Techniques</b> . . . . .	211
7.1 Introduction . . . . .	211
7.2 The Use of Ordinary Diffractometers . . . . .	211
7.3 Software and Hardware Requirements . . . . .	212
7.4 Available Instruments . . . . .	213
7.5 Selected Applications of a Portable X-ray Residual Stress Unit (By W. P. Evans) . . . . .	214
Reference . . . . .	229
<b>8 The Shape of Diffraction Peaks – X-ray Line Broadening</b> . . . . .	230
8.1 Introduction . . . . .	230
8.2 Slit Corrections . . . . .	233
8.3 Fourier Analysis of Peak Broadening . . . . .	238
Problem . . . . .	245
References . . . . .	247
<b>Appendix A: Solutions to Problems</b> . . . . .	248
<b>Appendix B</b> . . . . .	252
B.1 Introduction . . . . .	252
B.2 The Marion-Cohen Method . . . . .	252
B.3 Dölle-Hauk Method (Oscillation-free Reflections) . . . . .	254
B.4 Methods of Peiter and Lode . . . . .	256
B.5 Use of High Multiplicity Peaks . . . . .	257
References . . . . .	257
<b>Appendix C: Fourier Analysis</b> . . . . .	259
<b>Appendix D: Location of Useful Information in “International Tables for Crystallography”</b> . . . . .	266
<b>Appendix E: Values of <math>G_x</math> for Various Materials (By Dr. M. James)</b> . . . . .	267
<b>Appendix F: A Compilation of X-ray Elastic Constants (By Dr. M. James)</b> . . . . .	270
References . . . . .	271
<b>Subject Index</b> . . . . .	273