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in Mathematics  
and Its Applications**

**Volume 25**

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Avner Friedman   Willard Miller, Jr.

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# Solitons in Physics, Mathematics, and Nonlinear Optics

With 37 Illustrations



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Peter J. Olver  
School of Mathematics  
University of Minnesota  
Minneapolis, MN 55455, USA

David H. Sattinger  
School of Mathematics  
University of Minnesota  
Minneapolis, MN 55455, USA

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## FOREWORD

This IMA Volume in Mathematics and its Applications

### SOLITONS IN PHYSICS, MATHEMATICS, AND NONLINEAR OPTICS

is based on the proceedings of two workshops which were an integral part of the 1988-89 IMA program on NONLINEAR WAVES. The workshops focussed on the main parts of the theory of solitons and on the applications of solitons in physics, biology and engineering, with a special concentration on nonlinear optics. We thank the Coordinating Committee: James Glimm, Daniel Joseph, Barbara Keyfitz, Andrew Majda, Alan Newell, Peter Olver, David Sattinger and David Schaeffer for planning and implementing the stimulating year-long program. We especially thank the Workshop Organizers for Solitons in Physics and Mathematics, Alan Newell, Peter Olver, and David Sattinger, and for Nonlinear Optics and Plasma Physics, David Kaup and Yuji Kodama for their efforts in bringing together many of the major figures in those research fields in which solitons in physics, mathematics, and nonlinear optics theories are used.

Avner Friedman

Willard Miller, Jr.



## PREFACE

This volume includes some of the lectures given at two workshops, "Solitons in Physics and Mathematics" and "Solitons in Nonlinear Optics and Plasma Physics" held during the 1988-89 I.M.A. year on Nonlinear Waves. Since their discovery by Kruskal and Zabusky in the early 1960's, solitons have had a profound impact on many fields, ranging from engineering and physics to algebraic geometry. The present contributions represent only a fraction of these areas, but give the reader a good overview of several current research directions, including optics, fluid dynamics, inverse scattering, cellular automata, Backlund transformations, monodromy, Painlevé equations, symmetries and Hamiltonian systems.

It is a pleasure to thank the staff of the IMA and its directors, Avner Friedman and Willard Miller, Jr., for their professional handling of the workshops, which helped to promote many important interactions and new collaborations among participants. We would also like to thank Kaye A. Smith, Stephan J. Skogerboe and Patricia V. Brick for their help in the preparation of this volume.

Peter J. Olver  
David H. Sattinger

## CONTENTS

Foreword .....	ix
Preface .....	xi
Nonlinear evolution equations, inverse scattering and cellular automata.....	1
Mark J. Ablowitz	
Painlevé equations and the inverse scattering and inverse monodromy transforms.....	27
Mark J. Ablowitz	
Chaotic behavior in nonlinear polarization dynamics.....	45
D. David, D.D. Holm, and M.V. Tratnik	
New manifestations of solitons.....	65
A.S. Fokas	
A unified approach to recursion operators .....	79
A.S. Fokas and P.M. Santini	
Isospectral flows: their Hamiltonian structures, Miura maps and master symmetries.....	97
Allan P. Fordy	
Integrable nonlinear evolution equations in the description of waves in the shallow-water long-wave approximation.....	123
Decio Levi	
Transient solitons in stimulated Raman scattering .....	155
Curtis R. Menyuk	
Monodromy preserving deformation of linear ordinary and partial differential equations.....	165
Craig A. Tracy	
Bäcklund transformations and the Painlevé property.....	175
John Weiss	
Symmetry groups of multidimensional integrable nonlinear systems .....	203
P. Winternitz	