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Volume 26

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FOREWORD

This IMA Volume in Mathematics and its Applications

TWO PHASE FLOWS AND WAVES

is based on the proceedings of a workshop which was an integral part of the 1988-89 IMA program on NONLINEAR WAVES. The workshop focussed on the development of waves in flowing composites. We thank the Coordinating Committee: James Glimm, Daniel Joseph, Barbara Keyfitz, Andrew Majda, Alan Newell, Peter Olver, David Sattinger and David Schaeffer for planning and implementing the stimulating year-long program. We especially thank the Workshop Organizers, Daniel D. Joseph and David G. Schaeffer for their efforts in bringing together many of the major figures in those research fields in which modelling of granular flows and suspensions is used.

Avner Friedman

Willard Miller, Jr.

PREFACE

This Workshop, held from January 3-10, 1989 at IMA, focused on the properties of materials which consist of many small solid particles or grains. Let us distinguish the terms *granular material* and *suspension.* In the former, the material consists exclusively of solid particles interacting through direct contact with one another, either sustained frictional contacts in the case of slow shearing or collisions in the case of rapid shearing. In suspensions, also called two phase flow, the grains interact with one another primarily through the influence of a viscous fluid which occupies the interstitial space and participates in the flow. (As shown by the lecture of I. Vardoulakis (not included in this volume), the distinction between these two idealized cases is not always clear.) Both kinds of materials exhibit properties analogous to a fluid (they can flow freely and undergo large deformations) and analogous to a solid (e.g., a granular material can sustain some shearing stress at rest). Both kinds of flows have important industrial and geological applications; the fluidized beds used in many industrial processes deserve special emphasis in this connection.

These materials raise a number of new scientific questions. For the most part, the speakers at the Workshop used a continuum theory in their approaches to these problems. (However, the lecture of O. Walton (not included in this volume) considered a direct, particle-by-particle numerical simulation.) The majority of the issues studied revolve around the constitutive behavior of such materials. We are still far from a rigorous, complete derivation of continuum constitutive behavior. Several authors addressed this challenging problem. Others analyzed theoretically or solved numerically the partial differential equations which result when an *ad hoc* constitutive law is assumed; such a law may be based on experiment or on theoretical considerations. Other papers reported on experimental phenomena exhibited by such materials. Still others considered the application to fluidized beds.

The study of such materials is still at an early phase of its development, and the unresolved questions greatly outnumber the answered ones. These problems have been worked on by researchers from several different fields (engineering, physics, mathematics). In such an underdeveloped area it seems important to consider the perspectives of all these different fields. The purpose of the Workshop was to assist in combining these viewpoints by putting researchers in different fields in close contact with one another. If the lively discussions at the Workshop were a reliable indicator, the Workshop was very productive towards this end.

The contents of specific papers are as follows. First, regarding two phase flow, the papers of Drew, Arnold, and Lakey, of Jenkins and McTigue, and of Passman studied the fundamental problems of deriving constitutive behavior theoretically. Prosperetti and Satrape analyze the stability of various models for two phase flow, while Wallis analyzes one such model in detail. The papers of Gibilaro, Foscolo, and di Felice and of Singh and Joseph concern the important application of fluidized beds. Regarding granular flow, Baxter, Behringer, Fagert, and Johnson report on experiments. In separate papers, Collins and Schaeffer study mathematical properties of equations describing granular flow with an assumed constitutive law. Pitman

presents the results of the numerical solution of such equations. (Remark: It is noteworthy that no paper attempts to derive from micromechanics the constitutive behavior of a granular material in the friction-dominated slow flow regime. This absence is a reflection of the difficulty of the problem, not simply an oversight.)

There remains only the pleasant duty of thanking the lecturers for their stimulating contributions and the IMA staff for its courteous, efficient support of the Workshop.

Daniel D. Joseph
David G. Schaeffer

CONTENTS

Foreword viii
Preface ix

Pattern formation and time-dependence in flowing sand 1
G.W. Baxter, R.P. Behringer, T. Fagert, and G.A. Johnson

The mathematical structure of the equations for quasi-static plane strain deformations of granular material 30
Ian F. Collins

Relation of microstructure to constitutive equations 45
Donald A. Drew, Gary S. Arnold, and Richard T. Lahey, Jr.

The velocity of dynamic waves in fluidised beds 56
Larry Gibilaro, Piero Foscolo, and Renzo di Felice

Transport processes in concentrated suspensions: The role of particle fluctuations 70
James T. Jenkins and David F. McTigue

Stress in dilute suspensions 80
Stephen L. Passman

Computations of granular flow in a hopper 88
E. Bruce Pitman

Stability of two phase flow models 98
Andrea Prosperetti and James V. Satrape

Mathematical issues in the continuum formulation of slow granular flow 118
David G. Schaeffer

One-dimensional, particle bed models of fluidized suspensions ... 130
P. Singh and D.D. Joseph

On Geurst's equations for inertial coupling in two phase flow 150
Graham B. Wallis