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Coding Theory and Design Theory

Part I
Coding Theory

With 13 Illustrations



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FOREWORD

This IMA Volume in Mathematics and its Applications

Coding Theory and Design Theory Part I: Coding Theory

is based on the proceedings of a workshop which was an integral part of the 1987-88 IMA program on APPLIED COMBINATORICS. We are grateful to the Scientific Committee: Victor Klee (Chairman), Daniel Kleitman, Dijen Ray-Chaudhuri and Dennis Stanton for planning and implementing an exciting and stimulating year-long program. We especially thank the Workshop Organizer, Dijen Ray-Chaudhuri, for organizing a workshop which brought together many of the major figures in a variety of research fields in which coding theory and design theory are used.

Avner Friedman

Willard Miller, Jr.

PREFACE

Coding Theory and Design Theory are areas of Combinatorics which found rich applications of algebraic structures. Combinatorial designs are generalizations of finite geometries. Probably, the history of Design Theory begins with the 1847 paper of Reverend T.P. Kirkman "On a problem of Combinatorics", Cambridge and Dublin Math. Journal. The great Statistician R.A. Fisher reinvented the concept of combinatorial 2-design in the twentieth century. Extensive application of algebraic structures for construction of 2-designs (balanced incomplete block designs) can be found in R.C. Bose's 1939 Annals of Eugenics paper, "On the construction of balanced incomplete block designs". Coding Theory and Design Theory are closely interconnected. Hamming codes can be found (in disguise) in R.C. Bose's 1947 Sankhyā paper "Mathematical theory of the symmetrical factorial designs". The same paper also introduced the packing problem in projective spaces - the central problem in the construction of optimum linear codes. Coding theory has developed into a rich and beautiful example of abstract sophisticated mathematics being applied successfully to solve real-life problems of communication. Applications of deep theorems of Algebraic Geometry for construction of linear codes by V.D. Goppa and others created much excitement. Much work remains to be done to make the algebraic geometric codes practical and implementable. Theory of t -designs for $t > 2$ is in a state of rapid development. The 1987-88 Applied Combinatorics Program of IMA decided to devote the period from May 1, 1988 to June 25, 1988 to concentration on Design Theory and Coding Theory. It was particularly appropriate as many of the specialists that were invited worked in both of these areas.

The purpose of this section of the Applied Combinatorics Year was to bring together Coding Theorists, Design Theorists and Statisticians in the area of experimental designs, to exchange informations and ideas on the latest developments, to encourage interactions and to create an inspiring and stimulating research environment. This purpose was well served. Before the beginning of the workshops from May 1 to June 10, 1988 the pace was relaxed with plenty of time for research exchanges. During this period lectures of J.H. van Lint on Algebraic Geometric Codes was a particularly popular event. In this period there were also lectures by E. Assmus, R.A. Bailey, C-S. Cheng, M. Deza, A.S. Hedayat, S.L. Ma, V. Pless, D.K. Ray-Chaudhuri, N. Singhi, R.M. Wilson and L. Teirlinck. The periods of workshops, Coding Theory, June 13-17, 1988 and Design Theory, June 20-25, 1988 were much more intense with forty (40) lectures altogether. Symposium on Statistical theory of Experimental Designs attracted many statisticians with lively lectures by eight prominent statisticians. Most of the participants submitted their papers for publication in this volume on Coding Theory and Design Theory. Unfortunately a few fine lectures are not submitted for inclusion in these Proceedings.

Thanks are due to IMA director Professor A. Friedman, Associate director W. Miller, Jr. and IMA staff for their extremely helpful attitude and generous assistance. I take this opportunity to offer special thanks to Mrs. P. Brick, Mr. S. Skogerboe, and Mrs. K. Smith for their preparation of the manuscripts.

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