# The IMA Volumes in Mathematics and Its Applications

## Volume 20

*Series Editors*
Avner Friedman  Willard Miller, Jr.

# Institute for Mathematics and its Applications
## IMA

The **Institute for Mathematics and its Applications** was established by a grant from the National Science Foundation to the University of Minnesota in 1982. The IMA seeks to encourage the development and study of fresh mathematical concepts and questions of concern to the other sciences by bringing together mathematicians and scientists from diverse fields in an atmosphere that will stimulate discussion and collaboration.

The IMA Volumes are intended to involve the broader scientific community in this process.

Avner Friedman, Director
Willard Miller, Jr., Associate Director

* * * * * * * * * *

### IMA PROGRAMS

| | |
|---|---|
| 1982-1983 | **Statistical and Continuum Approaches to Phase Transition** |
| 1983-1984 | **Mathematical Models for the Economics of Decentralized Resource Allocation** |
| 1984-1985 | **Continuum Physics and Partial Differential Equations** |
| 1985-1986 | **Stochastic Differential Equations and Their Applications** |
| 1986-1987 | **Scientific Computation** |
| 1987-1988 | **Applied Combinatorics** |
| 1988-1989 | **Nonlinear Waves** |
| 1989-1990 | **Dynamical Systems and Their Applications** |
| 1990-1991 | **Phase Transitions and Free Boundaries** |

* * * * * * * * * *

### SPRINGER LECTURE NOTES FROM THE IMA:

*The Mathematics and Physics of Disordered Media*
Editors: Barry Hughes and Barry Ninham
(Lecture Notes in Math., Volume 1035, 1983)

*Orienting Polymers*
Editor: J.L. Ericksen
(Lecture Notes in Math., Volume 1063, 1984)

*New Perspectives in Thermodynamics*
Editor: James Serrin
(Springer-Verlag, 1986)

*Models of Economic Dynamics*
Editor: Hugo Sonnenschein
(Lecture Notes in Econ., Volume 264, 1986)

Dijen Ray-Chaudhuri

# Coding Theory and Design Theory

## Part I
## Coding Theory

With 13 Illustrations

Springer-Verlag
New York Berlin Heidelberg
London Paris Tokyo Hong Kong

Dijen Ray-Chaudhuri
Department of Mathematics
Ohio State University
Columbus, Ohio 43210
USA

*Series Editors*

Avner Friedman
Willard Miller, Jr.
Institute for Mathematics and Its Applications
University of Minnesota
Minneapolis, MN 55455 USA

Mathematical Subject Classification Codes:11T71, 94Bxx.

Library of Congress Cataloging-in-Publication Data

Coding theory and design theory / Dijen Ray-Chaudhuri, editor.
p. cm. — (The IMA volumes in mathematics and its applications ; v. 20–21)
"Based on the proceedings of a workshop which was an integral part of the 1987–88 IMA program on applied combinatorics"—Foreword.
Includes bibliographical references.
Contents: v. 1. Coding theory — v. 2. Design theory.
ISBN-13:978-1-4613-8996-5
1. Coding theory. 2. Experimental design. I. Ray-Chaudhuri, Dijen, 1933– II. Series.
QA268.C68 1990
003'.54—dc20 89-26336

Printed on acid-free paper.


ISBN-13:978-1-4613-8996-5/1989 $0.00 + 0.20

Camera-ready copy prepared by the IMA.

9 8 7 6 5 4 3 2 1

ISBN-13:978-1-4613-8996-5 e-ISBN-13:978-1-4613-8994-1
DOI: 10.1007/978-1-4613-8994-1

# The IMA Volumes
## in Mathematics and its Applications

### Current Volumes:

**Volume 1:** Homogenization and Effective Moduli of Materials and Media
Editors: Jerry Ericksen, David Kinderlehrer, Robert Kohn, J.-L. Lions

**Volume 2:** Oscillation Theory, Computation, and Methods of Compensated Compactness
Editors: Constantine Dafermos, Jerry Ericksen,
David Kinderlehrer, Marshall Slemrod

**Volume 3:** Metastability and Incompletely Posed Problems
Editors: Stuart Antman, Jerry Ericksen, David Kinderlehrer, Ingo Muller

**Volume 4:** Dynamical Problems in Continuum Physics
Editors: Jerry Bona, Constantine Dafermos, Jerry Ericksen, David Kinderlehrer

**Volume 5:** Theory and Applications of Liquid Crystals
Editors: Jerry Ericksen and David Kinderlehrer

**Volume 6:** Amorphous Polymers and Non-Newtonian Fluids
Editors: Constantine Dafermos, Jerry Ericksen, David Kinderlehrer

**Volume 7:** Random Media
Editor: George Papanicolaou

**Volume 8:** Percolation Theory and Ergodic Theory of Infinite Particle Systems
Editor: Harry Kesten

**Volume 9:** Hydrodynamic Behavior and Interacting Particle Systems
Editor: George Papanicolaou

**Volume 10:** Stochastic Differential Systems, Stochastic Control Theory and Applications
Editors: Wendell Fleming and Pierre-Louis Lions

**Volume 11:** Numerical Simulation in Oil Recovery
Editor: Mary Fanett Wheeler

**Volume 12:** Computational Fluid Dynamics and Reacting Gas Flows
Editors: Bjorn Engquist, M. Luskin, Andrew Majda

**Volume 13:** Numerical Algorithms for Parallel Computer Architectures
Editor: Martin H. Schultz

**Volume 14:** Mathematical Aspects of Scientific Software
Editor: J.R. Rice

**Volume 15:** Mathematical Frontiers in Computational Chemical Physics
Editor: D. Truhlar

**Volume 16:** Mathematics in Industrial Problems
by Avner Friedman

**Volume 17:** Applications of Combinatorics and Graph Theory to the Biological and Social Sciences
Editor: Fred Roberts

**Volume 18:** $q$-Series and Partitions
Editor: Dennis Stanton

**Volume 19:** Invariant Theory and Tableaux
Editor: Dennis Stanton

**Volume 20:** Coding Theory and Design Theory Part I: Coding Theory
Editor: Dijen Ray-Chaudhuri

**Volume 21:** Coding Theory and Design Theory Part II: Design Theory
Editor: Dijen Ray-Chaudhuri

**Volume 22:** Signal Processing: Part I
Editors: L. Auslander, F.A. Grünbaum, W. Helton, T. Kailath, P. Khargonekar and S. Mitter

**Volume 23:** Signal Processing: Part II
Editors: L. Auslander, F.A. Grünbaum, W. Helton, T. Kailath, P. Khargonekar and S. Mitter

**Volume 24:** Mathematics in Industrial Problems, Part 2
by Avner Friedman

**Forthcoming Volumes:**

1988-1989: *Nonlinear Waves*

Solitons in Physics and Mathematics

Two Phase Waves in Fluidized Beds, Sedimenation, and Granular Flows

Nonlinear Evolution Equations that Change Type
Computer Aided Proofs in Analysis
Multidimensional Hyperbolic Problems and Computations (2 Volumes)
Microlocal Analysis and Nonlinear Waves

Summer Program 1989: *Robustness, Diagnostics, Computing and Graphics in Statistics*
Robustness, Diagnostics in Statistics (2 Volumes)
Computing and Graphics in Statistics

1989-1990: *Dynamical Systems and Their Applications*
Introduction to Dynamical Systems

# FOREWORD

This IMA Volume in Mathematics and its Applications

**Coding Theory and Design Theory Part I: Coding Theory**

is based on the proceedings of a workshop which was an integral part of the 1987-88 IMA program on APPLIED COMBINATORICS. We are grateful to the Scientific Committee: Victor Klee (Chairman), Daniel Kleitman, Dijen Ray-Chaudhuri and Dennis Stanton for planning and implementing an exciting and stimulating year-long program. We especially thank the Workshop Organizer, Dijen Ray-Chaudhuri, for organizing a workshop which brought together many of the major figures in a variety of research fields in which coding theory and design theory are used.

Avner Friedman

Willard Miller, Jr.

## PREFACE

Coding Theory and Design Theory are areas of Combinatorics which found rich applications of algebraic structures. Combinatorial designs are generalizations of finite geometries. Probably, the history of Design Theory begins with the 1847 paper of Reverand T.P. Kirkman "On a problem of Combinatorics", Cambridge and Dublin Math. Journal. The great Statistician R.A. Fisher reinvented the concept of combinatorial 2-design in the twentieth century. Extensive application of algebraic structures for construction of 2-designs (balanced incomplete block designs) can be found in R.C. Bose's 1939 Annals of Eugenics paper, "On the construction of balanced incomplete block designs". Coding Theory and Design Theory are closely interconnected. Hamming codes can be found (in disguise) in R.C. Bose's 1947 Sankhyā paper "Mathematical theory of the symmetrical factorial designs". The same paper also introduced the packing problem in projective spaces - the central problem in the construction of optimum linear codes. Coding theory has developed into a rich and beautiful example of abstract sophisticated mathematics being applied successfully to solve real-life problems of communication. Applications of deep theorems of Algebraic Geometry for construction of linear codes by V.D. Goppa and others created much excitement. Much work remains to be done to make the algebraic geometric codes practical and implementable. Theory of $t$-designs for $t > 2$ is in a state of rapid development. The 1987-88 Applied Combinatorics Program of IMA decided to devote the period from May 1, 1988 to June 25, 1988 to concentration on Design Theory and Coding Theory. It was particularly appropriate as many of the specialists that were invited worked in both of these areas.

The purpose of this section of the Applied Combinatorics Year was to bring together Coding Theorists, Design Theorists and Statisticians in the area of experimental designs, to exchange informations and ideas on the latest developments, to encourage interactions and to create an inspiring and stimulating research environment. This purpose was well served. Before the beginning of the workshops from May 1 to June 10, 1988 the pace was relaxed with plenty of time for research exchanges. During this period lectures of J.H. van Lint on Algebraic Geometric Codes was a particularly popular event. In this period there were also lectures by E. Assmus, R.A. Bailey, C-S. Cheng, M. Deza, A.S. Hedayat, S.L. Ma, V. Pless, D.K. Ray-Chaudhuri, N. Singhi, R.M. Wilson and L. Teirlinck. The periods of workshops, Coding Theory, June 13–17, 1988 and Design Theory, June 20–25, 1988 were much more intense with forty (40) lectures altogether. Symposium on Statistical theory of Experimental Designs attracted many statisticians with lively lectures by eight prominent statisticians. Most of the participants submitted their papers for publication in this volume on Coding Theory and Design Theory. Unfortunately a few fine lectures are not submitted for inclusion in these Proceedings.

Thanks are due to IMA director Professor A. Friedman, Associate director W. Miller, Jr. and IMA staff for their extremely helpful attitude and generous assistance. I take this opportunity to offer special thanks to Mrs. P. Brick, Mr. S. Skogerboe, and Mrs. K. Smith for their preparation of the manuscripts.

## CONTENTS — PART I

## CONTENTS — PART II