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Introduction to Arithmetical Functions

Springer-Verlag
New York Berlin Heidelberg Tokyo

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AMS Classifications: 10-01, 10A20, 10A21, 10H25

Library of Congress Cataloging-in-Publication Data
McCarthy, Paul J. (Paul Joseph)
Introduction to arithmetical functions.
(Universitext)
Bibliography: p.
Includes index.
1. Arithmetic functions. I. Title.
QA245.M36 1985 512'.7 85-26068

With 6 illustrations.

9 8 7 6 5 4 3 2 1

ISBN-13: 978-0-387-96262-7 e-ISBN-13: 978-1-4613-8620-9
DOI: 10.1007/978-1-4613-8620-9

Preface

The theory of arithmetical functions has always been one of the more active parts of the theory of numbers. The large number of papers in the bibliography, most of which were written in the last forty years, attests to its popularity. Most textbooks on the theory of numbers contain some information on arithmetical functions, usually results which are classical. My purpose is to carry the reader beyond the point at which the textbooks abandon the subject. In each chapter there are some results which can be described as contemporary, and in some chapters this is true of almost all the material.

This is an introduction to the subject, not a treatise. It should not be expected that it covers every topic in the theory of arithmetical functions. The bibliography is a list of papers related to the topics that are covered, and it is at least a good approximation to a complete list within the limits I have set for myself. In the case of some of the topics omitted from or slighted in the book, I cite expository papers on those topics.

Each chapter is followed by notes which are bibliographical in nature, and only incidentally historical. My purpose in the notes is to point out sources of results. Number theory, and the theory of arithmetical functions in particular, is rife with rediscovery, so I hope the reader will not be too harsh with me if I fail to pin down the truly first source of some result. Perhaps this book will help reduce the rate of rediscovery.

There are more than four hundred exercises. They form an essential part of my development of the subject, and during any serious reading of the book some time must be spent thinking about the exercises.

I assume that the reader is familiar with calculus, including infinite series, and has the maturity gained from completing several mathematics courses at the college level. A first course in the theory of numbers provides more than enough background in number theory. In fact, only a few things from such a course are used, such as elementary properties of congruences and the unique factorization theorem.

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