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Volume 67

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Avner Friedman Willard Miller, Jr.

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Preface

This is the seventh volume in the series “Mathematics in Industrial Problems.” The motivation for these volumes is to foster interaction between Industry and Mathematics at the “grass roots level;” that is, at the level of specific problems. These problems come from Industry: they arise from models developed by the industrial scientists in ventures directed at the manufacture of new or improved products. At the same time, these problems have the potential for mathematical challenge and novelty.

To identify such problems, I have visited industries and had discussions with their scientists. Some of the scientists have subsequently presented their problems in the IMA Seminar on Industrial Problems. The book is based on the seminar presentations and on questions raised in subsequent discussions. Each chapter is devoted to one of the talks and is self-contained. The chapters usually provide references to the mathematical literature and a list of open problems which are of interest to the industrial scientists. For some problems a partial solution is indicated briefly. The last chapter of the book contains a short description of solutions to some of the problems raised in previous volumes, as well as references to papers in which such solutions have been published.

The speakers in the Seminar on Industrial Problems have given us at the IMA hours of delight and discovery. My thanks to David K. Lambert (General Motors Research and Development), David S. Ross (Eastman Kodak), Leonard J. Gray (Oak Ridge National Laboratory), Kevin R.J. Ellwood (Ford Motor Company), Lee A. Feldkamp (Ford Motor company), Hamid R. Mortazavi (3M), Rita T. Farouki (IBM Thomas J. Watson Research Center), Shih-Ping Liou (Siemens Corporate Research), Leonid Gurvitz (Siemens Corporate Research), Blaise Morton (Honeywell), Michael L. Honig (Bellcore), David K. Misemer (3M), Marius Orłowski (Motorola), Allen Cox (Honeywell), Jack F. Douglas (National Institute for Standard and Technology), Dennis Perchak (Eastman Kodak), Ronald Mahler (UNISYS), Leonard Borucki (Motorola), Edward P. Furlani (Eastman Kodak) and Zhenhua Mao (Motorola).

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Director
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and its Applications
July 10, 1994

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