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Introductory Problem Courses in Analysis and Topology

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Preface to the Teacher

In each chapter of this book, the student finds definitions, and theorems which are guaranteed to be true. The student's job is to prove the theorems. In the problems that follow, various propositions are stated as if they were true, but many of them turn out to be false. Here the student's job is to find out the truth, whatever it may be, and establish it with a proof or a counter-example.

Any adequate explanation of the workings and the advantages of problem courses would form an essay much too long for a preface. Following are some observations.

--Any student who has creative capacities deserves an opportunity to discover, use, and develop them; and this opportunity should come soon. It should not be postponed until the time comes to write a dissertation.

--In the first few weeks of a problem course, the teacher is likely to see striking evidence of the need for such a course. At the beginning, it rarely happens that a student can write grammatical sentences which say what he really means. At the end of a problem course, almost all students can do this.

--In a lecture course it is taken for granted that whatever the teacher says is right, except for occasional lapses. If at some point the logic of a proof is less than clear, then faith and authority may take over where reason left off. But students are aware that students are often wrong, and therefore they do not accept one another's proofs unless they understand the proofs completely. This establishes the critical vigilance that the student needed all along, and it sets a new standard of thoroughness of comprehension.

--Some have supposed that problem courses are advantageous only for students of real brilliancy, but my own experience over many years indicates the contrary. The time that is "wasted" while students grope their way makes the pace of a problem course very slow. (It often happens that a whole hour is spent analyzing a "proof" which turns out to be quite worthless.) This means that a competent student is able to keep track, and finds at the end that he understands the course completely. This is a valuable experience, and for many students it is new.

The choice of material, in the courses presented here, is based on two different considerations.

I believe that "baby real-variable theory" is so fundamental that it needs to be over-learned. To examine its technical apparatus, carefully, once, is not enough. This apparatus needs to be absorbed so completely that it forms, forever after, part of the student's intuition. Some of this material is exciting, but some of it is dull. Personally, I can tolerate almost any sort of spade-work, if there is good reason to do it, and if I am doing it myself; but to watch carefully while somebody else does it is a much worse hardship. I doubt that this attitude is unusual .... For these two reasons, I think that a problem course is a good medium for learning the beginnings of analysis.

Elementary set-theoretic topology has a different advantage. It seems an ideal gymnasium for learning what one might call Applied Mathematical Logic. In this material, anti-intuitive results are remarkably prevalent. Thus the tiniest logical slip often leads the student to "prove" a proposition which is false. When this happens, it is plain that logical rigor is not something imposed on the student by academic authority; it is an objective necessity.

Both these courses are short, and so any student who really wants to know something about analysis and topology is going to have to take other courses in order to do so. This was intentional. I believe that every undergraduate should take at least one problem course, and probably two; but this style of study should not be unduly prolonged. If it is, it may lead to consequences which are both unnecessary and sad: the student may come to feel that nothing is worth knowing unless you discover it for yourself, so that study is an activity inappropriate for a thinker. Being aware that this state of mind is a handicap, I prefer not to propagate it.

Edwin E. Moise

New York, November, 1981

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