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**Essential Mathematics for
Applied Fields**

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Preface

1. Purpose

The purpose of this work is to provide, in one volume, a wide spectrum of essential (non-measure theoretic) Mathematics for use by workers in the variety of applied fields. To obtain the background developed here in one volume would require studying a prohibitive number of separate Mathematics courses (assuming they were available). Before, much of the material now covered was (a) unavailable, (b) too widely scattered, or (c) too advanced as presented, to be of use to those who need it. Here, however, we present a sound basis requiring only Calculus through Differential Equations. It provides the needed flexibility to cope, in a rigorous manner, with the every-day, non-standard and new situations that present themselves. There is no substitute for this.

2. Arrangement

The volume consists of twenty Sections, falling into several natural units:

Basic Real Analysis

1. Sets, Sequences, Series, and Functions
2. Doubly Infinite Sequences and Series
3. Sequences and Series of Functions
4. Real Power Series
5. Behavior of a Function Near a Point: Various Types of Limits
6. Orders of Magnitude: the 0 , o , \sim Notation
7. Some Abelian and Tauberian Theorems

Riemann-Stieltjes Integration

8. 1-Dimensional Cumulative Distribution Functions and Bounded Variation Functions
9. 1-Dimensional Riemann-Stieltjes Integral
10. n-Dimensional Cumulative Distribution Functions and Bounded Variation Functions
11. n-Dimensional Riemann-Stieltjes Integral

The Finite Calculus

12. Finite Differences and Difference Equations

Basic Complex Analysis

13. Complex Variables

Applied Linear Algebra

14. Matrices and Determinants
15. Vectors and Vector Spaces
16. Linear Equations and Generalized Inverse
17. Characteristic Roots and Related Topics

Miscellaneous

18. Convex Sets and Convex Functions
19. Max-Min Problems
20. Some Basic Inequalities

3. Development

Each Section develops its topic rigorously, based upon material previously established; that is, it is self-contained. Throughout the body of the text, at appropriate locations, are found solved Examples and Exercises requiring solution; both are critical parts of the development. Complete Hints or Answers are provided for the Exercises following each Section, as are References to Additional and Related Material.

A serious attempt has been made to include essential Mathematics, and to allow the References to provide entry into the vast literature that exists. Accordingly, it should be noted that Section 13 develops Complex Variables only through Elementary Contour Integration, but enough theory is developed to pursue more advanced topics.

4. Use

The volume can be used as a basis of a one or two semester course covering some or all of the topics. It can be used to supplement existing courses by serving as a remedial reference when deficiencies are noticed. It can also be used in an independent (or guided) study plan. It is also a source for review, or entry into more advanced and/or related literature.

5. Acknowledgements

During the ten years within which this work matured into its present form, many individuals at the State University of New York at Buffalo, and elsewhere, have helped. It was appreciated. Carefully preserved lecture notes taken at the University at Chapel Hill provided useful Exercises and Examples at various points in the text.

Rosanna Bello of Buffalo did the final typescript during a long six-month period of cooperation. Inger (Tulle) Abbott of Williamsville was the artist who rendered the diagrams and drafting details. Professor Steven L. Siegel of Niagara University aided in final proofreading, suggestions, and indexing. Their cooperation, and Helen Meyer's understanding, speeded completion

of the project under otherwise adverse conditions.

Finally, I note the freedom and cooperation extended to me by Springer-Verlag through their representatives, Mathematics Editor Kaufmann-Bühler and Editorial Assistant Jane Walsh.

R.M.M.

Williamsville, New York

June 8, 1979

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