Applications of Interval Computations

Applied Optimization

Volume 3

The titles published in this series are listed at the end of this volume.

Applications of Interval Computations

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PREFACE

Primary Audience for the Book

- Specialists in numerical computations who are interested in algorithms with automatic result verification.
- Engineers, scientists, and practitioners who desire results with automatic verification and who would therefore benefit from the experience of successful applications.
- Students in applied mathematics and computer science who want to learn these methods.

Goal Of the Book

This book contains surveys of applications of interval computations, i.e., applications of numerical methods with automatic result verification, that were presented at an international workshop on the subject in El Paso, Texas, February 23–25, 1995. The purpose of this book is to disseminate detailed and surveyed information about existing and potential applications of this new growing field.

Brief Description of the Papers

At the most fundamental level, interval arithmetic operations work with sets: The result of a single arithmetic operation is the set of all possible results as the operands range over the domain. For example, $[0.9, 1.1] + [2.9, 3.1] = [3.8, 4.2]$, where $[3.8, 4.2] = \{x + y | x \in [0.9, 1.1] \text{ and } y \in [3.8, 4.2]\}$. The power of interval arithmetic comes from the fact that (i) the elementary operations and standard functions can be computed for intervals with formulas and subroutines; and (ii) *directed roundings* can be used, so that the images of these operations (e.g.

[3.8, 4.2] in the preceding example) *rigorously* contain the *exact* result of the computations. These facts allow

- rigorous enclosure of roundoff error, truncation error, and errors in data;
- computation of rigorous bounds on the ranges of functions.

In fact, interval arithmetic is a convenient and effective means of obtaining information used in place of Lipschitz constants. Such information, combined with bounds on the ranges of functions that interval arithmetic supplies, is widely recognized as valuable in algorithms for global optimization. In "A Review of Techniques in the Verified Solution of Constrained Global Optimization Problems," Kearfott reviews the literature, including several books on the subject, outlines some of the basic techniques, including some of his own experimental ones, and gives advice.

A related fundamental problem, the numerical analysis of linear systems of equations, is treated in the articles by Alefeld et al. and by Rohn. Interval arithmetic computations, naively arranged, sometimes implicitly assume independent variation among quantities among which there actually are relationships. This interval dependency leads to overestimation (i.e. lack of sharpness) in computed solution sets. For example, handling a symmetric linear system with uncertainty in the coefficients as an arbitrary, unsymmetric system leads to such interval dependency. In "The Shape of the Symmetric Solution Set," Alefeld, Kreinovich and Mayer describe the exact solution set to symmetric linear systems as an intersection of linear and quadratic inequalities. In "Linear Interval Equations: Computing Enclosures with Bounded Relative Or Absolute Overestimation is NP-Hard," Jiří Rohn shows that there is no polynomial time algorithm that computes bounds on the solution sets to *all* interval linear systems with overestimation less than δ, for any $\delta > 0$. This result can guide algorithm developers. Fortunately, heuristic methods such as interval Gaussian elimination or the interval Gauss–Seidel method work well for many systems in practice, when the widths of the coefficient intervals are small and also in many other cases.

Hadjihassan et al. and Jerrell use these fundamental numerical analysis considerations in manufacturing design and in economic input-output models, respectively. In both works, uncertainties in the input data are encompassed by describing them as intervals. In "Quality Improvement via the Optimization of Tolerance Intervals During the Design Stage," Hadjihassan, Walter and Pronzato use a combination of interval and non-interval techniques to optimize the tolerance of manufactured items, subject to uncertainties in the inputs. In "Applications of Interval Computations to Regional Economic Input-Output Models," Jerrell effectively uses fundamental properties of interval linear systems in Leontief input/output models. Jerrell thus obtains meaningful bounds for the impact of Northern Arizona University on the economy of Coconino County, Arizona.

Related to global optimization, computational existence and uniqueness proofs are possible with interval computations. Fefferman and Seco's contribution, "Interval Arithmetic in Quantum Mechanics" is an example of a computer-assisted proof outside of general global optimization algorithms. Fefferman and Seco discuss the role of interval arithmetic in establishment of an inequality related to a precise asymptotic formula for the ground state of a non-relativistic atom.

Also related to global optimization is ***constraint propagation***, increasingly popular in recent years in fields such as robotics and computer-aided geometric design, to verify intersection or non-intersection, etc. Several authors have also applied the technique in general global optimization and nonlinear equation systems codes and software. In this technique, sometimes implemented in languages such as Prolog, relationships among intermediate quantities in arithmetic expressions in constraints are used recursively to compute ever-narrower bounds on solution variables. This volume contains two software systems that allow such computations. In "Interval Computations on the Spreadsheet," Hyvönen and De Pascale explain an extension of Microsoft Excel that allows interval computations, and constraint propagation in particular. In "Solving Optimization Problems with Help of the UniCalc Solver," Semenov describes an integrated interactive environment, available commercially, that uses subdefinite programming, related to interval constraint propagation, to solve various types of optimization problems.

Experts in statistics and in fuzzy logic have found the ability of intervals to represent uncertainties valuable. Various higher-order structures have been defined and utilized. In "Automatically Verified Arithmetic on Probability Distributions and Intervals," Berleant surveys statistical applications of interval computations, while the surveys "Nested Intervals and Sets: Concepts, Relations to Fuzzy Sets, and Applications" by Nguyen and Kreinovich, "Fuzzy Interval Inference Utilizing the Checklist Paradigm and BK-Relational Products" by Kohout and Bandler, and "Computing Uncertainty in Interval Based Sets" by Rocha, Kreinovich, and Kearfott describe applications to fuzzy logic and expert systems.

In the past, widespread application of interval computations has been limited by availability of hardware and software support. Also, experts in interval computations have urged the development of faster, hardware-oriented systems. In "Software and Hardware Techniques for Accurate, Self-Validating Arithmetic," Schulte and Swartzlander review some of the available programming language extensions, packages and problem solving environments for interval computations. They then discuss hardware design alternatives and propose a specific hardware design and software interface for variable precision interval arithmetic. In "Stimulating Hardware and Software Support for Interval Arithmetic," Walster presents his view of why many hardware and software vendors do not presently provide interval computations as integral parts of their products. He then discusses steps the community of interval computations experts should take to make such capabilities more widely available.

Finally, our introduction gives a more detailed elementary explanation of the fundamentals of interval arithmetic, as well as lengthier elementary explanations of the surveys that follow.

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R. Baker Kearfott and Vladik Kreinovich
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