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PREFACE

The goal of the research out of which this monograph grew, was to make annealing as much as possible a general purpose optimization routine. At first glance this may seem a straight-forward task, for the formulation of its concept suggests applicability to any combinatorial optimization problem. All that is needed to run annealing on such a problem is a unique representation for each configuration, a procedure for measuring its quality, and a neighbor relation. Much more is needed however for obtaining acceptable results consistently in a reasonably short time. It is even doubtful whether the problem can be formulated such that annealing becomes an adequate approach for all instances of an optimization problem. Questions such as what is the best formulation for a given instance, and how should the process be controlled, have to be answered. Although much progress has been made in the years after the introduction of the concept into the field of combinatorial optimization in 1981, some important questions still do not have a definitive answer.

In this book the reader will find the foundations of annealing in a self-contained and consistent presentation. Although the physical analogue from which the concept emanated is mentioned in the first chapter, all theory is developed within the framework of markov chains. To achieve a high degree of instance independence adaptive strategies are introduced. Much emphasis is on these topics throughout the book, and in the last chapter a pascal implementation of these strategies is included. Every fact used in that implementation is proven in the preceding chapters. The book is therefore at the same time an introduction into annealing and its applications, a compendium for the theoretical background of annealing, a basis for further research, and a report on the progress made in developing a multi-purpose annealing routine. A recipe for state space construction cannot be

given. Even an efficient analysis of the space regarding its adequacy for annealing has not yet appeared. But the aspects of state space construction are thoroughly discussed and illustrated.

Annealing may finally not end up being a general purpose optimization routine, but it will certainly remain a useful tool. It is easy to implement for a specific application (and hopefully this book makes access to this method even easier), and such an implementation is quite insensitive to the exact nature of the object function if only reasonable solutions are required. This is important, for example when developing a large software system, where representative solutions are needed for trying other parts of the systems, and experiments with different object functions are desirable before undertaking the much more intensive task of implementing a deterministic optimization routine specific for a certain object function. In practice, annealing has already proven its usefulness, and several gigantic design automation programs have been outperformed by the conceptually simple annealing procedure.

Most of the original research reported here has been done at the Thomas J. Watson Research Center of the IBM Corporation in Yorktown Heights, NY. That research center was also the place where annealing as a combinatorial optimization tool was conceived. Especially, Dan Gelatt's efforts to search for possible applications provided an early exposure of the concept to the authors. Many people, also outside the IBM Company, have directly and indirectly contributed to the maturing of the ideas presented in this book. These are gratefully acknowledged. This book has been prepared for printing by *Witan Presentaties*, using the \LaTeX document preparation system.

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