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Many-Particle Physics

SECOND EDITION

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Preface

This textbook is for a course in advanced solid-state theory. It is aimed at graduate students in their third or fourth year of study who wish to learn the advanced techniques of solid-state theoretical physics. The method of Green's functions is introduced at the beginning and used throughout. Indeed, it could be considered a book on practical applications of Green's functions, although I prefer to call it a book on physics. The method of Green's functions has been used by many theorists to derive equations which, when solved, provide an accurate numerical description of many processes in solids and quantum fluids. In this book I attempt to summarize many of these theories in order to show how Green's functions are used to solve real problems. My goal, in writing each section, is to describe calculations which can be compared with experiments and to provide these comparisons whenever available.

The student is expected to have a background in quantum mechanics at the level acquired from a graduate course using the textbook by either L. I. Schiff, A. S. Davydov, or I. Landau and E. M. Lifshitz. Similarly, a prior course in solid-state physics is expected, since the reader is assumed to know concepts such as Brillouin zones and energy band theory. Each chapter has problems which are an important part of the lesson; the problems often provide physical insights which are not in the text. Sometimes the answers to the problems are provided, but usually not. It is hoped that the student can learn the subject by using the book as a study guide, since small enrollments often restrict the availability of courses at the advanced level.

I am often asked why I wrote this book. The questioner usually has an understanding of the work involved and so regards my effort as reflecting poorly upon my sanity. On the whole I agree, and if I knew at the beginning

the actual investment of time, I probably would not have started. The actual time is about four to five hours per page, which is roughly divided equally among writing, editing, and library searching. My reason for undertaking this project was the great need for someone to do it and the disinclination of anyone else to write a comprehensive advanced textbook on solid-state theory. My own graduate students kept asking me for the standard reference on a number of topics, and I became tired of replying that none exist. My objective was to take standard subjects, such as the electron gas or polaron theory, and to summarize what is generally known. All the steps are retained in the derivation, so that the answers are obtained by starting from the beginning and working through to the end.

The volume is restricted to a description of the many-particle theory of solids. There is also some discussion of quantum fluids, which have historically been part of solid-state theory. The important subject of classical fluids is omitted entirely. In solids and liquids, the forces between pairs of particles are well understood, and the starting Hamiltonian for the problem is accurate. Here we are better off than our brethren in nuclear or stellar physics, since they are often groping for the Hamiltonian. In solids, the only problem is that there are usually 10^{23} particles in the system. Thus we have a well-defined many-body problem which is easy to state: We have simple forces between particles, and the only complication is the large number of particles. In this regard, the atomic theorists have an easier task since they usually have fewer than one hundred electrons in their theoretical system. Consequently, they have been more successful in achieving a quantitative description of atoms. However, solid-state theory has a richer variety of phenomena: magnetism, superconductivity, superfluidity, phase transitions, etc. We have been successful in describing most of these phenomena with great accuracy—magnetism is probably the greatest exception. There is no doubt that solid-state theory has been the center of developments in many-body theory, and our successes are followed by exporting these ideas to other disciplines; e.g., the plasma theory of metals becomes the giant dipole resonance of nuclei.

The topics chosen for discussion were selected on the basis of what every well-rounded theorist should know. Thus basic subjects such as the electron gas, electron-phonon interactions, transport theory, linear response, superconductivity, and superfluidity are covered. Other subjects were deemed equally important, but I ran out of energy, and the book became too long anyway; the important subjects which were omitted are the Kondo effect, Hubbard models, Anderson models, and magnetic systems. Also omitted were two important subjects which deserve large textbooks of

their own: renormalization group and the Hohenberg–Kohn–Sham theory of the inhomogeneous electron gas.

Anyone writing an advanced book is going to receive some criticism. No one is expert in all advanced subjects. There are many topics on which I am not an expert and not even well informed. In this situation any choice of action will be criticized. If the topic is omitted, I am criticized for regarding it as unimportant. If it is included, then I am criticized for not providing the expert viewpoint. The only solution is to do one's best and to challenge critics to write their own book.

Historians of science have described numerous and competing models for how science advances. One model is the Ortega hypothesis which suggests that science advances by a large number of mediocre scientists each making a small incremental contribution. On the opposite end of the spectrum is a quite different view that science advances by great leaps forward by the intellectual giants such as Newton and Einstein. The rest of us merely fill in the details they overlook. In preparing this manuscript, I became aware that solid-state theory has advanced by a process intermediate between these extremes. In each chapter, usually six or eight theorists seem to dominate the subject and to provide most of the major concepts and understanding. But in the next chapter, on another subject, it is usually another entirely different group of six to eight theorists who provide the progress. Thus the advance of science appears to occur by a large number of talented workers, although in each topic only a few are important.

Quite often in the text it is necessary to evaluate standard integrals from tables. I use *Table of Integrals, Series and Products* by I. S. Gradshteyn and I. M. Ryzhik (Academic Press, New York, 1965). It seems to be available worldwide. All special integrals are referred to in this present book as "G & R" followed by the integral number.

It is a pleasure to thank many associates for the substantial assistance I have received in the preparation of the book. About half of the writing was done while I was on leave, from Indiana University, as visiting Professor at Chalmers University of Technology in Gothenburg, Sweden. I wish to thank Professor A. Sjölander and S. Lundqvist for this stimulating and very pleasant year. My financial support was provided by the Nordic Institute for Theoretical Astrophysics (NORDITA) in Copenhagen, and I wish to thank Professors A. Bohr, A. Luther, and J. W. Wilkins for arranging my visiting professorship. The entire draft was read by S. M. Girvin, who deserves special thanks for catching many lapses in the first draft. Additional proofreading was also provided by M. Jonson and P. Tua.

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Indiana University

Gerald D. Mahan

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