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**BINARY DECISION DIAGRAMS  
AND APPLICATIONS  
FOR VLSI CAD**

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by

**Shin-ichi Minato**

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Kanagawa, Japan

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# FOREWORD

Symbolic Boolean manipulation using Binary Decision Diagrams (BDDs) has been applied successfully to a wide variety of tasks, particularly in very large scale integration (VLSI) computer-aided design (CAD). The concept of decision graphs as an abstract representation of Boolean functions dates back to early work by Lee and Akers. In the last ten years, BDDs have found widespread use as a concrete data structure for symbolic Boolean manipulation. With BDDs, functions can be constructed, manipulated, and compared by simple and efficient graph algorithms. Since Boolean functions can represent not just digital circuit functions, but also such mathematical domains as sets and relations, a wide variety of CAD problems can be solved using BDDs.

Although I can claim some credit in initiating the use of BDDs for symbolic Boolean manipulation, the state of the art in the field has been advanced by researchers around the world. In particular, the group headed by Prof. Shuzo Yajima at Kyoto University has been the source of many important research results as well as the spawning ground for some of the most productive researchers. Shin-Ichi Minato is a prime example of this successful research environment. While a Master's student at Kyoto University, he and his colleagues developed important refinements to the BDD data structure, including a shared representation, attributed edges, and improved variable ordering techniques. Since joining the NTT LSI Laboratories, Minato has continued to make important research contributions. Perhaps most significant among these is the use of a zero-suppressed reduction rule when using BDDs to represent sparse sets. These "ZBDDs" have proved effective for such tasks as the cube-set manipulations in two-level logic optimization, as well as for representing polynomial expressions.

This book is based on Minato's Phd dissertation and hence focuses on his particular contributions to the field of BDDs. The book provides valuable information for both those who are new to BDDs as well as to long time aficionados. Chapters 1–4 provide a thorough and self-contained background to the area, in part because Minato's contributions have become part of the mainstream practice. Chapters 5–7 present an area for which Minato's contributions have been of central importance, namely the encoding of combinatorial problems as sparse sets and their solution using ZBDDs. Chapters 8–9 carry these ideas beyond Boolean functions, using BDDs to



represent polynomial expressions and integer-valued functions. These latter parts provide intriguing insights into how BDDs can be applied outside VLSI CAD.

Randal E. Bryant  
Carnegie Mellon University

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# PREFACE

This book is a revised edition of my doctoral thesis, submitted to Kyoto University, Japan, in 1995. It is mainly a report of my research on the techniques of Boolean function manipulation using *Binary Decision Diagrams (BDDs)* and their applications for VLSI CAD systems. This research was done in Prof. Yajima's Laboratory at Kyoto University from 1987 to 1990 and in NTT LSI Laboratories from 1990 to 1995.

Over the past ten years, since appearance of Bryant's paper in 1986, BDDs have attracted the attention of many researchers because of their suitability for representing Boolean functions. They are now widely used in many practical VLSI CAD systems. I hope that this book can serve as an introduction to BDD techniques and that it presents several new ideas on BDDs and their applications. I expect many computer scientists and engineers will be interested in this book since Boolean function manipulation is a fundamental technique not only in digital system design but also in exploring various problems in computer science.

The contributions of this book are summarized as follows. Chapters 2 through 4 present implementation and utility techniques of BDDs. Currently, there are few books or papers concisely describing the basic method of BDD manipulation, so those chapters can be read as an introduction to BDDs.

Transforming BDDs into another data structure is an important issue. Chapter 5 proposes a fast method for generating compact cube sets from BDDs. Cube set data structure is commonly used in digital system design, and the proposed method will find many applications since the cube sets are also used for representing a kind of knowledge in artificial intelligence or data base systems.

The concept of *Zero-suppressed BDDs (ZBDDs)*, introduced in Chapter 6, is the most important idea in this book. The use of ZBDDs results in the implicit cube set representation described in Chapter 7, and it greatly accelerates multi-level logic synthesis systems and enlarges the scale of the circuits to which these systems are applicable. Chapter 8 provides another application of ZBDDs: the manipulation of arithmetic polynomial formulas. This chapter presents the newest topic, not included in the original doctoral thesis.

Chapter 9 can be read independently from the other chapters. Here we apply the BDD techniques for solving various problems in computer science, such as the 8-queens problem, the traveling salesman problem, and a kind of scheduling problems. We developed an arithmetic Boolean expression manipulator, which is a helpful tool for the research or education related to discrete functions. This program, runs on a SPARC station, is open to the public. Anyone interested in this program can get it from an anonymous FTP server authorized by the *Information Processing Society of Japan*:  
eda.kuee.kyoto-u.ac.jp (130.54.29.134) /pub/cad/BemII.

Throughout I have assumed that the reader of this book is a researcher, an engineer, or a student who is familiar with switching theory. Since BDD is a graph-based representation, I have endeavored to provide plentiful illustrations to help the reader's understanding, and I have shown experimental results whenever possible, in order to demonstrate the practical side of my work.

As always, this work would not have been possible without the help of many other people. First I would like to express my sincere appreciation to Professor Shuzo Yajima of Kyoto University for his continuous guidance, interesting suggestions, and encouragement during this research. I would also like to express my thanks to Dr. Nagisa Ishiura of Osaka University, who introduced me to the research field of VLSI CAD and BDDs, and has been giving me invaluable suggestions, accurate criticism, and encouragement throughout this research.

I express my deep gratitude to Professor Randal E. Bryant of Carnegie Mellon University. I started this research with his papers, and I received valuable suggestions and comments from him. I am honored by his writing the Foreword to this book.

I also acknowledge interesting comments that I have received from Professor Hiromi Hiraishi of Kyoto Sangyo University, Professor Hiroto Yasuura of Kyushuu University, and Associate Professor Naofumi Takagi of Nagoya University. I would like to thank Dr. Kiyoharu Hamaguch, Mr. Koich Yasuoka, and Mr. Shoichi Hirose of Kyoto University, Associate Professor Hiroyuki Ochi of Hiroshima City University, Mr. Yasuhito Koumura of Sanyo Corporation, and Kazuya Ioki of IBM Japan Corporation for their interesting discussions and for their collaboration in implementing the BDD program package.

Special thanks are also due to Professor Saburo Muroga of the University of Illinois at Urbana-Champaign and Professor Tsutomu Sasao of Kyushu Institute of Technology for their instructive comments and advice on the publication of this book. I am grateful to Dr. Masahiro Fujita and Mr. Yusuke Matsunaga of Fujitsu Corporation, who started to work on BDDs early. I referred to their papers many times, and had fruitful discussions with them.

I also wish to thank Dr. Olivier Coudert of Synopsys corporation for his discussions on the implicit set representation, which led to the idea of ZBDDs. I would like to thank Professor Gary D. Hachtel and Associate Professor Fabio Somenzi of the University of Colorado at Boulder for their fruitful discussions on the application of BDDs to combinatorial problems. I am also grateful to Professor Robert K. Brayton of the University of California at Berkeley, Dr. Patrik McGeer of Cadance Berkeley Laboratories, Dr. Yoshinori Watanabe of Digital Equipment Corporation, and Mr. Yuji Kukimoto of the University of California at Berkeley for their interesting discussions and suggestions on the techniques of BDD-based logic synthesis.

Thanks are also due to my colleagues in NTT Research Laboratories. I gratefully acknowledge the collaboration with Dr. Toshiaki Miyazaki and Dr. Atsushi Takahara in developing and evaluating the BEM-II program. I am also grateful to Mr. Hiroshi G. Okuno and Mr. Masayuki Yanagiya for their interest in BEM-II applications. I also thank Mr. Noriyuki Takahashi for his helpful comments on the application of ZBDDs for fault simulation.

I wish to express my special gratitude to Mr. Yasuyoshi Sakai, Dr. Osamu Karatsu, Mr. Tamio Hoshino, Mr. Makoto Endo, and all the other members of NTT Advanced LSI Laboratory for giving me an opportunity to write this book and for their advice and encouragement. Thanks are due to all the members of the Professor Yajima's Laboratory for their discussions and helpful supports throughout this research.

Lastly, I thank my parents and my wife for their patience, support, and encouragement.

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