

Electron and Ion Optics

MICRODEVICES

Physics and Fabrication Technologies

Series Editors: Ivor Brodie and Julius J. Muray

*SRI International
Menlo Park, California*

ELECTRON AND ION OPTICS

Miklos Szilagy

GaAs DEVICES AND CIRCUITS

Michael Shur

SEMICONDUCTOR LITHOGRAPHY

Principles, Practices, and Materials

Wayne M. Moreau

A Continuation Order Plan is available for this series. A continuation order will bring delivery of each new volume immediately upon publication. Volumes are billed only upon actual shipment. For further information please contact the publisher.

Electron and Ion Optics

Miklos Szilagyı

*University of Arizona
Tucson, Arizona*

Plenum Press • New York and London

Library of Congress Cataloging in Publication Data

Szilágyi, Miklós.

Electron and ion optics / Miklós Szilágyi.

p. cm. — (Microdevices)

Bibliography: p.

Includes index.

ISBN-13: 978-1-4612-8247-1

e-ISBN-13: 978-1-4613-0923-9

DOI: 10.1007/978-1-4613-0923-9

1. Electron optics. 2. Electron beams. 3. Ion bombardment. 4. Electromagnetic lenses.

I. Title. II. Series.

QC793.5.E62S95 1988

537.5'6—dc19

87-32180

CIP

© 1988 Plenum Press, New York

Softcover reprint of the hardcover 1st edition 1988

A Division of Plenum Publishing Corporation

233 Spring Street, New York, N.Y. 10013

All rights reserved

No part of this book may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, microfilming, recording, or otherwise, without written permission from the Publisher

To Jutka, Gabor, and Zoltan
For the time that should have belonged to them

Preface

The field of electron and ion optics is based on the analogy between geometrical light optics and the motion of charged particles in electromagnetic fields. The spectacular development of the electron microscope clearly shows the possibilities of image formation by charged particles of wavelength much shorter than that of visible light.

As new applications such as particle accelerators, cathode ray tubes, mass and energy spectrometers, microwave tubes, scanning-type analytical instruments, heavy beam technologies, etc. emerged, the scope of particle beam optics has been extended to the formation of fine probes. The goal is to concentrate as many particles as possible in as small a volume as possible.

Fabrication of microcircuits is a good example of the growing importance of this field. The current trend is towards increased circuit complexity and pattern density. Because of the diffraction limitation of processes using optical photons and the technological difficulties connected with x-ray processes, charged particle beams are becoming popular. With them it is possible to write directly on a wafer under computer control, without using a mask. Focused ion beams offer especially great possibilities in the submicron region. Therefore, electron and ion beam technologies will most probably play a very important role in the next twenty years or so.

Many books have been published on different aspects of the theory and applications of electron and ion optics. This one is intended to be a self-contained, systematic and *up-to-date* introduction to the field. The reader can find the derivation of the most essential relationships, the understanding of which is absolutely necessary to do any meaningful work with particle beams and optics, but which are usually presented without proof in most books. *Modern computer methods are especially emphasized.* The book can serve as a textbook for engineers, scientists, and graduate students who wish to understand the basic principles of electron and ion optics and apply them to the design and/or operation of beam-type devices and instruments.

The author is well aware of the fact that it is not easy to reach these goals. One volume is hardly adequate even to cover the theoretical foundations and most important applications. Therefore, the treatment is restricted to the presentation of the basics and the most recent results of research, including the author's own. Throughout the book the relevance of the presented material to practical applications is emphasized, but no attempt has been made to review the applications themselves. We have given over 400 basic references, including

publications that report on recent developments in the field. Owing to limitations of space, the list of references is very far from being complete.

To study this book no previous acquaintance with physical electronics is required. The necessary introductory information is presented in Chapter 1. The theoretical material is derived from basic principles. Because of the mathematical nature of the subject, however, a thorough knowledge of calculus (including vector calculus) is presupposed.

The author has been working in this field since 1958. It was a great joy for me to be able to sit down and summarize the basics of my knowledge in the present volume. This knowledge has been accumulated in the course of interaction with numerous people. I am especially grateful to Professors V. M. Kelman, S. Ya. Yavor, and A. D. Sushkov and to the memory of the unforgettable Dennis Gabor. My research work in the last three years has been supported by the National Science Foundation. I appreciate the continuous support of Dean T. Triffet of the University of Arizona, the inspiration of the editors of this series, and the high-quality work of Plenum Press. The book would have never been completed without the encouragement, patience, and love of my family.

Miklos Szilagyi

Tucson, Arizona

Contents

CHAPTER 1. Introductory Survey

1-1. Introduction	1
1-2. Electromagnetic Fields	2
1-2-1. Maxwell's Equations	2
1-2-2. Static Fields	4
1-2-3. Stokes's Theorem	5
1-3. Some Basic Classical Mechanics	5
1-3-1. Hamilton's Principle; The Lagrangian Equations of Motion	6
1-3-2. The Maupertuis Principle	8
1-4. A Little Reminder of Geometrical Optics	8
1-4-1. Fermat's Principle; The Index of Refraction	9
1-4-2. Axially Symmetric Lenses	9
Summary	12

CHAPTER 2. Motion of Charged Particles in Electric and Magnetic Fields

2-1. The Lagrangian	13
2-2. Conservation of Energy	16
2-2-1. Motion of Free Particles; Velocity versus Potential	17
2-3. The Equations of Motion	19
2-4. The Trajectory Equations	24
2-5. The Relativistic Potential	28
2-6. The Electron Optical Index of Refraction	29
2-7. Particles in Homogeneous Fields	31
2-7-1. The Parallel-Plate Capacitor	31
2-7-1-1. Electrostatic Deflection	34
2-7-1-2. A Simple Velocity Analyzer	35
2-7-2. Homogeneous Magnetic Field	36
2-7-2-1. Long Magnetic Lens	39
2-7-2-2. Magnetic Deflection	40
2-7-3. The Simultaneous Action of Homogeneous Electric and Magnetic Fields	41
2-7-3-1. Mass Analysis and Other Applications	45

2-8. Scaling Laws	47
Summary	50

CHAPTER 3. Determination of Electric and Magnetic Fields

3-1. Analytical Methods	52
3-1-1. Series Expansions of Potentials and Fields	52
3-1-1-1. Planar Fields	56
3-1-1-2. Axially Symmetric Fields	58
3-1-1-3. Multipole Fields	60
3-1-2. Analytical Calculation of Axially Symmetric Potential Fields	67
3-1-2-1. Separation of Variables	67
3-1-2-2. Difficulties of Analytical Calculations (Electrostatic Field of Two Equidiameter Cylinders)	72
3-1-2-3. Field of a Circular Aperture	77
3-1-2-4. Rapid Evaluation of Fields Produced by Two or More Circular Apertures	83
3-1-3. Analytical Calculation of Multipole Fields	86
3-1-3-1. Short Multipoles	86
3-1-3-2. Long Multipoles	87
3-1-3-3. Ideal Multipoles	91
3-1-3-4. The Method of Conformal Transformation	94
3-1-4. On the Role of Magnetic Materials	96
3-1-5. Analytical Calculation of Magnetic Fields Produced by Currents	102
3-1-5-1. The Biot–Savart Law	102
3-1-5-2. Field of a Straight Wire	104
3-1-5-3. Field of a Circular Loop	104
3-1-5-4. Field of a Thin Solenoid	105
3-1-5-5. Field of a Multilayer Coil	106
3-1-5-6. Field of a Pancake Coil	108
3-2. Measurement of Fields and Analog Methods	109
3-2-1. Measurement of Magnetic Fields	109
3-2-1-1. Electromagnetic Induction	110
3-2-1-2. Hall Effect	110
3-2-1-3. Permalloy and Bismuth Probes	111
3-2-1-4. Magnetic Resonance	111
3-2-2. Analog Methods	112
3-2-2-1. The Electrolytic Tank	112
3-2-2-2. The Resistor Network	114
3-2-2-3. Other Analog Methods	119
3-3. Numerical Methods	119
3-3-1. Accuracy	119
3-3-1-1. Errors Due to the Nature of the Problem	120
3-3-1-2. Errors Due to the Number Representation in the Com- puter	121
3-3-1-3. Errors Due to the Numerical Method	122
3-3-2. The Finite-Difference Method	122
3-3-2-1. Methods of Solution for Systems of Algebraic Equations	128

3-3-3. The Finite-Element Method	130
3-3-4. The Charge-Density (Integral) Method	137
3-3-5. Numerical Differentiation and Interpolation	143
3-3-5-1. Differentiation	143
3-3-5-2. Lagrange Interpolation	145
3-3-5-3. The Interpolating Pulse	146
3-3-5-4. The Cubic Spline	147
Summary	149

CHAPTER 4. Focusing With Axially Symmetric Fields

4-1. Busch's Theorem	151
4-2. The General Trajectory Equation	153
4-3. The Paraxial Ray Equation	156
4-4. Image Formation by Paraxial Rays	162
4-5. The Helmholtz–Lagrange Formula	164
4-6. Cardinal Elements	166
4-6-1. Asymptotic Cardinal Elements	170
4-7. Electron and Ion Lenses	176
4-8. Systems of Lenses	178
4-8-1. The Transfer Matrix	178
4-8-2. Combination of Two Thick Lenses	181
4-9. The Thin-Lens Approximation	186
4-9-1. Combination of Thin Lenses	191
4-10. Examples of Paraxial Focusing	194
4-10-1. Paraxial Trajectories in Homogeneous Fields	194
4-10-1-1. Homogeneous Electrostatic Field	194
4-10-1-2. Skew Rays	196
4-10-1-3. Homogeneous Magnetic Field	197
4-10-2. The Single-Loop Magnetic Lens	200
4-10-3. Lens Systems	203
4-10-3-1. Telescopic System	203
4-10-3-2. Magnification of Lens Systems	204
Summary	206

CHAPTER 5. The Theory of Aberrations

5-1. The Method of Characteristic Functions	208
5-2. Geometrical Aberrations	216
5-2-1. Spherical Aberration	222
5-2-1-1. Zero and Infinite Magnifications	227
5-2-1-2. Alternative Forms of the Spherical Aberration Coefficient	229
5-2-1-3. Scherzer's Theorem	236
5-2-1-4. The Disk of Minimum Confusion	237
5-2-2. Astigmatism	240
5-2-3. Curvature of Field	241
5-2-4. Distortion	242

5-2-5. Coma	244
5-2-6. Anisotropic Aberrations	245
5-2-6-1. Anisotropic Astigmatism	246
5-2-6-2. Anisotropic Distortion	247
5-2-6-3. Anisotropic Coma	248
5-2-7. On the Relative Importance of the Different Geometrical Aberrations	250
5-3. Chromatic Aberration	251
5-3-1. Axial Chromatic Aberration	255
5-3-1-1. Zero and Infinite Magnifications	258
5-3-1-2. The Upper Limit of the Axial Chromatic Aberration	259
5-3-2. Chromatic Aberration of Magnification	262
5-3-3. Anisotropic Chromatic Aberration	262
5-3-4. Magnetic Chromatic Aberration	263
5-4. Asymptotic Aberrations	264
5-4-1. The Dependence of the Asymptotic Aberration Coefficients on the Magnification	266
5-4-1-1. Polynomial Expression for the Asymptotic «Spherical Aberration Coefficient	267
5-4-1-2. Polynomial Expression for the Asymptotic Axial Chromatic Aberration Coefficient	271
5-4-2. Aberrations of Thin Lenses	274
5-4-2-1. Spherical Aberration	274
5-4-2-2. Axial Chromatic Aberration	275
5-5. Aberrations of Lens Combinations	276
5-5-1. Addition of Spherical Aberrations	277
5-5-2. Addition of Axial Chromatic Aberrations	280
5-6. Other Sources of Aberrations and Aberration Correction	281
5-6-1. Diffraction	282
5-6-2. Space Charge and Surface Charges	283
5-6-3. High-Frequency Fields	284
5-6-4. Lack of Axial Symmetry	284
5-6-5. Other Methods of Correction	285
5-6-5-1. Coaxial Lenses	285
5-6-5-2. Symmetric Trajectories	285
5-6-5-3. Position of the Limiting Aperture	285
5-6-5-4. Digital Image Processing	286
5-6-6. Synthesis	286
5-6-7. On the Measurement of Aberrations	286
5-6-8. Brightness	286
5-7. Simultaneous Action of Different Aberrations	289
5-7-1. Negligibly Small Sources	290
5-7-2. Finite Sources	291
5-7-2-1. Negligible Chromatic Aberration	292
5-7-2-2. Negligible Spherical Aberration	294
5-7-3. Aberration Mixing for Lens Combinations	295
5-7-4. Figures of Merit	295
Summary	297

CHAPTER 6. Numerical Techniques for Ray Tracing and Calculation of Aberrations

6-1. Analytical Models	299
6-2. Numerical Ray Tracing	300
6-2-1. The Runge–Kutta Method	302
6-2-2. Multistep Methods	306
6-2-2-1. Numerov’s Method	307
6-2-3. Additional Remarks on Accuracy	307
6-3. Numerical Calculation of Aberration Integrals	308
6-3-1. Trapezoidal Integration	309
6-3-2. Simpson’s Rule	310
6-3-3. Romberg Integration and the Gaussian Quadrature	311
Summary	312

CHAPTER 7. Electrostatic Lenses

7-1. General Properties and Relationships	313
7-2. Electrostatic Lens Models	316
7-2-1. Analytical Models	316
7-2-2. The Piecewise Linear Model	316
7-2-3. The Piecewise Quadratic Model	317
7-2-4. The Spline Model	320
7-3. Two-Electrode Immersion Lenses	320
7-3-1. Geometrically Symmetric Lenses	322
7-3-1-1. A Linear Model	322
7-3-1-2. An Analytical Model	326
7-3-1-3. The Two-Cylinder Lens	331
7-3-1-4. The Double-Aperture Lens	343
7-3-1-5. Polynomial Lenses	344
7-3-2. Asymmetric Lenses	348
7-3-2-1. Analytical Models	348
7-3-2-2. The Asymmetric Two-Cylinder Lens	350
7-3-2-3. A Hybrid Lens	350
7-4. Unipotential Lenses	352
7-4-1. Symmetric Lenses	355
7-4-1-1. A Piecewise Linear Model	355
7-4-1-2. A Piecewise Quadratic Model	359
7-4-1-3. An Analytical Model	360
7-4-1-4. The Three-Cylinder Lens	364
7-4-1-5. The Triple-Aperture Lens	369
7-4-1-6. Other Types of Symmetric Lenses	369
7-4-2. Asymmetric Lenses	370
7-5. Three-Electrode Immersion Lenses	373
7-5-1. Geometrically Symmetric Lenses	373
7-5-1-1. The Three-Cylinder Lens	374
7-5-1-2. Other Types of Geometrically Symmetric Lenses	379
7-5-2. Asymmetric Lenses	379

7-6. Multielectrode Lenses	380
7-6-1. Four-Electrode Lenses	380
7-6-2. Lenses with Five or More Electrodes	382
7-6-3. Spline Lenses	383
7-7. Comparison of Different Electrostatic Lenses	384
7-8. Lenses Immersed in Fields	385
7-8-1. The Exponential Model	385
7-8-2. The Single-Aperture Lens	386
7-8-3. Cathode Lenses, Electron and Ion Sources	388
7-8-3-1. Thermionic Guns	390
7-8-3-2. Field-Emission Guns	391
7-8-3-3. Ion Sources	392
Summary	393

CHAPTER 8. Magnetic Lenses

8-1. General Properties and Relationships	395
8-2. Long Lenses	397
8-2-1. Homogeneous Magnetic Fields	397
8-2-2. Linear Magnetic Fields	398
8-2-3. Long Lenses with Low Spherical Aberration	401
8-3. Magnetic Lens Models	401
8-3-1. The Rectangular Model	401
8-3-2. The Step-Function Model	401
8-3-3. The Piecewise Linear Model	402
8-3-4. The Spline Model	402
8-3-5. Glaser's Bell-Shaped Model	402
8-3-5-1. Generalization of the Bell-Shaped Model	410
8-3-6. The Grivet-Lenz Model	411
8-3-7. Other Models	412
8-4. Short Lenses	413
8-4-1. Conventional Lenses	413
8-4-2. Unconventional Lenses	419
8-4-2-1. Superconducting Lenses	419
8-4-2-2. Reduction of the Coil Size by Other Means	419
8-4-2-3. Rotation-Free Miniature Lenses	419
8-4-2-4. Iron-Free Magnetic Lenses	420
8-4-2-5. Single Pole-Piece Lenses	420
Summary	421

CHAPTER 9. Computer-Aided Optimization and Synthesis of Electron and Ion Lenses

9-1. Is Aberrationless Electron/Ion Optics Possible?	423
9-1-1. The Lower Limit of the Axial Chromatic Aberration of Magnetic Lenses	426
9-2. Optimization: Synthesis versus Analysis	426

9-3. Early Attempts of Synthesis	428
9-4. Calculus of Variations	429
9-4-1. The Lower Limits of the Spherical and Axial Chromatic Aberration Coefficients	432
9-5. Dynamic Programming	434
9-6. Optimal Control Procedure	440
9-7. Analytical Functions	441
9-8. Reconstruction of Electrodes and Pole Pieces from the Optimized Axial Field Distributions	442
9-9. Polynomial and Spline Lenses	446
9-9-1. Polynomial Lenses	446
9-9-2. Spline Lenses	447
9-9-2-1. Two-Interval Spline Lenses	449
9-10. The Synthesis Procedure	453
9-10-1. Application: Unconventional Electrostatic Lenses	456
9-11. Artificial Intelligence Techniques	459
Summary	459

CHAPTER 10. Multipole Lenses

10-1. The Fields of Multipole Lenses	461
10-2. The Paraxial Ray Equations	464
10-3. Image Formation by Paraxial Rays	466
10-4. Systems of Quadrupoles	468
10-4-1. Transfer Matrices	468
10-4-2. Thin-Lens Representation	469
10-4-3. Doublets	471
10-4-4. Triplets	474
10-4-5. Multiplets	475
10-4-5-1. Beam Matching	475
10-5. Aberrations of Multipole Lenses	476
10-5-1. Geometrical Aberrations	476
10-5-2. Correction of Aberrations by Means of Multipoles	477
10-5-3. Chromatic Aberration	478
10-5-3-1. The Achromatic Quadrupole Lens	478
Summary	479

CHAPTER 11. Beam Deflection

11-1. Deflection for Scanning	481
11-1-1. Electrostatic Deflection Fields	481
11-1-2. Magnetic Deflection Fields	484
11-1-3. Stigmatic Imaging with Small Deflection	485
11-1-4. Deflection Aberrations	487
11-2. Electrostatic and Magnetic Prisms	490
11-2-1. Electrostatic Prisms	490
11-2-2. Magnetic Prisms	493

11-3. New Symmetries—New Possibilities	494
Summary	495
CHAPTER 12. High-Intensity Beams	
12-1. Space-Charge Optics	497
12-1-1. Space-Charge Forces	499
12-1-1-1. The Electrostatic Force	499
12-1-1-2. The Magnetic Force	501
12-1-2. Beam Spreading	502
12-1-3. Production of High-Intensity Beams	505
12-1-3-1. Space-Charge Flow	505
12-1-3-2. The Pierce Gun	506
12-1-4. Maintenance of High-Intensity Beams	508
12-1-4-1. Focusing by Homogeneous Magnetic Fields	508
12-1-4-2. Periodic Focusing	510
12-2. The Boersch Effect	512
Summary	512
References	515
Index	529