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Serge Lang

Short Calculus

The Original Edition of
"A First Course in Calculus"

With 30 Illustrations

Springer

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Foreword

The First Course in Calculus went through five editions since the early sixties. Sociological and educational conditions have evolved in various ways during four decades. Hence it has been found worth while to make the original edition again available. It is also worth while repeating here most of the foreword which I wrote almost forty years ago.

The purpose of a first course in Calculus is to teach the student the basic notions of derivative and integral, and the basic techniques and applications which accompany them.

At present in the United States, this material is covered mostly during the first year of college. Ideally, the material should be taught to students who are approximately sixteen years of age, and belongs properly in the secondary schools. (I have talked with several students of that age, and find them perfectly able to understand what it is all about.)

Irrespective of when it is taught, I believe that the presentation remains more or less invariant. The very talented student, with an obvious aptitude for mathematics, will rapidly require a course in functions of one real variable, more or less as it is understood by professional mathematicians. This book is not primarily addressed to such students (although I hope they will be able to acquire from it a good introduction at an early age).

I have not written this course in the style I would use for an advanced monograph, on sophisticated topics. One writes an advanced monograph for oneself, because one wants to give permanent form to one's vision of some beautiful part of mathematics, not otherwise accessible, somewhat in the manner of a composer setting down his symphony in musical notation.

This book is written for the student, to provide an immediate, and pleasant, access to the subject. I hope that I have struck a proper compromise between dwelling too much on special details, and not giving enough technical exercises, necessary to acquire the desired familiarity with the subject. In any case, certain routine habits of sophisticated mathematicians are unsuitable for a first course.

This does not mean that so-called rigour has to be abandoned. The logical development of the mathematics of this course from the most basic axioms proceeds through the following stages:

Set theory
Integers (whole numbers)
Rational numbers (fractions)
Numbers (i.e. real numbers)
Limits
Derivatives
and forward.

No one in his right mind suggests that one should begin a course with set theory. It happens that the most satisfactory place to jump into the subject is between limits and derivatives. In other words, any student is ready to accept as intuitively obvious the notions of numbers and limits and their basic properties. For some reason, there is a fashion which holds that the best place to enter the subject logically is between numbers and limits. Experience shows that the students do *not* have the proper psychological background to accept this, and resist it tremendously. Of course, there is still another fashion, which is to omit proofs completely. This does not teach mathematics, and puts students at a serious disadvantage for subsequent courses, and the understanding of what goes on.

In fact, it turns out that one can have the best of all these ideas. The arguments which show how the properties of limits can be reduced to those of numbers form a self-contained whole. Logically, it belongs *before* the subject matter of our course. Nevertheless, we have inserted it as an appendix. If any students feel the need for it, they need but read it and visualize it as Chapter 0. In that case, everything that follows is as rigorous as any mathematician would wish it (so far as objects which receive an analytic definition are concerned). Not one word need be changed in any proof. I hope this takes care once and for all of possible controversies concerning so-called rigour.

Some objects receive a geometric definition, and there are applications to physical concepts. In that case, it is of course necessary to insert one step to bridge the physical notion and its mathematical counterpart. The major instances of this are the functions sine and cosine, and the area, as an integral.

For sine and cosine, we rely on the notions of plane geometry. If one accepts standard theorems concerning plane figures then our proofs satisfy the above-mentioned standards.

For the integral, we first give a geometric argument. We then show, using the usual Riemann sums, how this geometric argument has a perfect counterpart when we require the rules of the game to reduce all definitions and proofs to numbers. This should satisfy everybody. Furthermore, the theory of the integral is so presented that only its existence depends either on a geometric argument or a slightly involved theoretical investigation (upper and lower sums). According to the level of ability of a class, the teacher may therefore dose the theory according to ad hoc judgement.

It is not generally recognized that some of the major difficulties in teaching mathematics are analogous to those in teaching a foreign language. (The secondary schools are responsible for this. Proper training in the secondary schools could entirely eliminate this difficulty.) Consequently, I have made great efforts to carry the student verbally, so to say, in using proper mathematical language. Some proofs are omitted. For instance, they would be of the following type. In the theory of maxima and minima, or increasing and decreasing functions, we carry out in full just one of the cases. The other is left as an exercise. The changes needed in the proof are slight, amounting mainly to the insertion of an occasional minus sign, but they force students to understand the situation and train them in writing clearly. This is very valuable. Aside from that, such an omission allows the teacher to put greater emphasis on certain topics, if necessary, by carrying out the other proof. As in learning languages, repetition is one of the fundamental tools, and a certain amount of mechanical learning, as distinguished from logical thinking, is both healthy and necessary.

I have made no great innovations in the exposition of calculus. Since the subject was discovered some 300 years ago, it was out of the question. Rather, I have omitted some specialized topics which no longer belong in the curriculum. Stirling's formula is included only for reference, and can be skipped, or used to provide exercises. Taylor's formula is proved with the integral form of the remainder, which is then properly estimated. The proof with integration by parts is more natural than the other (differentiating some complicated expression pulled out of nowhere), and is the one which generalizes to the higher dimensional case. I have placed integration after differentiation, because otherwise one has no technique available to evaluate integrals. But on the whole, everything is fairly standard.

I have cut down the amount of analytic geometry to what is both necessary and sufficient for a general first course in this type of mathematics. For some applications, more is required, but these applications are fairly specialized. For instance, if one needs the special properties concerning the focus of a parabola in a course on optics, then that is the place to present them, not in a general course which is to serve mathematicians, physicists, chemists, biologists, and engineers, to mention but a few. What is important is that the basic idea of representing a graph by a figure in the plane should be thoroughly understood, together with basic examples. The more abstruse properties of ellipses, parabolas, and hyperbolas should be skipped.

As for the question: why republish a forty year old edition? I answer:

Because for various reasons, a need exists for a short, straightforward and clear introduction to the subject. Adding various topics may be useful in some respects, and adding more exercises also, but such additions may also clutter up the book, especially for students with no or weak background.

To conclude, if I may be allowed another personal note here, I learned how to teach the present course from Artin, the year I wrote my Doctor's thesis. I could not have had a better introduction to the subject.

SERGE LANG

New Haven, 2002

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