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The Optimal Design of Blocked and Split-Plot Experiments



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Preface

Quality has become an important source of competitive advantage for the modern company. Therefore, quality control has become one of its key activities. Since the control of existing products and processes only allows moderate quality improvements, the optimal design of new products and processes has become extremely important. This is because the flexibility, which characterizes the design stage, allows the quality to be built in products and processes. In this way, substantial quality improvements can be achieved.

An indispensable technique in the design stage of a product or a process is the statistically designed experiment for investigating the effect of several factors on a quality characteristic. A number of standard experimental designs like, for instance, the factorial designs and the central composite designs have been proposed. Although these designs possess excellent properties, they can seldom be used in practice. One reason is that using standard designs requires a large number of observations and can therefore be expensive or time-consuming. Moreover, standard experimental designs cannot be used when both quantitative and qualitative factors are to be investigated or when the factor levels are subject to one or more constraints. A danger inherent to the use of standard designs is that the experimental situation is adapted to the experimental design available. Of course, it is much better to find the best possible design for the experimental situation at hand. This is exactly the purpose of the approach known as the optimal design of experiments.

In this book, the optimal design approach is applied to two common types of experiments, namely blocked and split-plot experiments. Blocked experiments are needed when not all the experimental observations can be carried out under homogeneous circumstances, for example when more than one batch of material is required or when the experiment takes up more than one day. Split-plot experiments are used when it is impractical to change the levels of some of the experimental factors. A typical example of such factor is temperature because heating a furnace and cooling it down are time-consuming operations. Often, the observations in blocked and split-plot experiments are correlated. This statistical dependence is explicitly taken into account in the optimal design approach as well.

Chapter 1 contains an overview of the experimental design literature. Special attention is given to the basics of the optimal design approach, as well as to the standard response surface designs and to categorical designs. Chapter 2 focuses on the optimal design of experiments with nonhomogeneous error variance and correlated observations. The topic of blocking experiments receives attention as well. In Chapter 3, the design problems considered in this book are described in detail and an appropriate statistical model is introduced. In Chapter 4, optimal designs for blocked experiments are computed. In Chapter 5, the optimal design approach is applied to a blocked optometry experiment. In the Chapters 6, 7 and 8, the optimal design of split-plot experiments is considered. Chapter 9 provides a brief overview of the recent results on two-level factorial and fractional factorial designs. Finally, Chapter 10 summarizes the main results of this book as well as some ideas for future research.

For every type of experiment considered in this book, an algorithm for the computation of optimal designs was developed. A Fortran 77 implementation of the algorithms as well as a number of sample in- and output files can be downloaded from the author's personal website <http://www.econ.kuleuven.ac.be/peter.goos/>. In case this link fails, the author websites on <http://www.springer-ny.com/> will provide an alternative.

While finishing this book, I would like to express my appreciation to those who have contributed to its development and its improvement. Firstly, I would like to thank my advisor Professor M. Vandebroek for her continuing assistance and support during the past few years. I am also indebted to the other members of my doctoral committee, the Professors P. Darius, E. Demeulemeester, A. Donev and W. Gochet, for their invaluable comments and suggestions. I am also grateful to the Fund for Scientific Research-Flanders (Belgium) for providing me with the financial means to carry out the research described in this book, to the Department of Applied Economics of the Katholieke Universiteit Leuven for providing me with a comfortable office and to my colleagues for creating an agreeable working

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