

# **Lecture Notes in Statistics**

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Erkki P. Liski  
Nripes K. Mandal  
Kirti R. Shah  
Bikas K. Sinha

# Topics in Optimal Design



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Erkki P. Liski  
Department of Mathematics, Statistics,  
and Philosophy  
University of Tampere  
Kehruukoulunkatu 1  
Tampere 33014  
Finland

Nripes K. Mandal  
Department of Statistics  
Calcutta University  
35, B.C. Road  
Calcutta 700 019  
India

Kirti R. Shah  
Department of Statistics  
and Actuarial Science  
University of Waterloo  
Waterloo, Ontario N2L 3G1  
Canada

Bikas K. Sinha  
Stat-Math Divison  
Indian Statistical Institute  
203 Barrackpore Trunk Road  
Calcutta 700 035  
India

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# Preface

In the early nineties, at the initiative of Sinha and financial support of Shah and Liski (from their respective Research Project Funds), the authors – inspired by their similar research interests – started collaborative research at various institutions mostly in pairs and triplets. It took more time and efforts on the part of Mandal to visit the others at regular intervals and keep track of their common as well as diverse research areas and merge his own. From this collaborative work, the concept of this monograph took a preliminary shape only last year and serious efforts were started to combine diverse avenues into one.

Admittedly, it took more time than expected to converge to a common platform regarding the contents and broad coverage of the topics to be included. We were mostly guided by our own common research interests spanning over the last ten years. That covered optimal designs in both discrete and continuous settings.

Availability of huge published literature in various statistical journals on the broad theme of optimal designs has made our task quite interesting and stimulating. We hope our readers will be as excited and delighted to read the monograph as we have been in our efforts to write it.

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June 2001

Erkki P. Liski  
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Kirti R. Shah  
Bikas K. Sinha

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Bikas K. Sinha

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