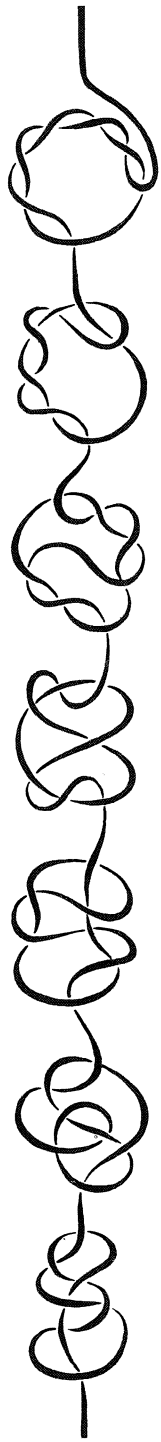
Graduate Texts in Mathematics

57

Richard H. Crowell
Ralph H. Fox

Introduction to Knot Theory



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To the memory of

Richard C. Blanchfield and Roger H. Kyle

and

RALPH H. FOX

Preface to the Springer Edition

This book was written as an introductory text for a one-semester course and, as such, it is far from a comprehensive reference work. Its lack of completeness is now more apparent than ever since, like most branches of mathematics, knot theory has expanded enormously during the last fifteen years. The book could certainly be rewritten by including more material and also by introducing topics in a more elegant and up-to-date style. Accomplishing these objectives would be extremely worthwhile. However, a significant revision of the original work along these lines, as opposed to writing a new book, would probably be a mistake. As inspired by its senior author, the late Ralph H. Fox, this book achieves qualities of effectiveness, brevity, elementary character, and unity. These characteristics would be jeopardized, if not lost, in a major revision. As a result, the book is being republished unchanged, except for minor corrections. The most important of these occurs in Chapter III, where the old sections 2 and 3 have been interchanged and somewhat modified. The original proof of the theorem that a group is free if and only if it is isomorphic to $F[\mathscr{A}]$ for some alphabet $\mathscr{A}$ contained an error, which has been corrected using the fact that equivalent reduced words are equal.

I would like to include a tribute to Ralph Fox, who has been called the father of modern knot theory. He was indisputably a first-rate mathematician of international stature. More importantly, he was a great human being. His students and other friends respected him, and they also loved him. This edition of the book is dedicated to his memory.

Richard H. Crowell

Dartmouth College
1977

Preface

Knot theory is a kind of geometry, and one whose appeal is very direct because the objects studied are perceivable and tangible in everyday physical space. It is a meeting ground of such diverse branches of mathematics as group theory, matrix theory, number theory, algebraic geometry, and differential geometry, to name some of the more prominent ones. It had its origins in the mathematical theory of electricity and in primitive atomic physics, and there are hints today of new applications in certain branches of chemistry.[1] The outlines of the modern topological theory were worked out by Dehn, Alexander, Reidemeister, and Seifert almost thirty years ago. As a subfield of topology, knot theory forms the core of a wide range of problems dealing with the position of one manifold imbedded within another.

This book, which is an elaboration of a series of lectures given by Fox at Haverford College while a Philips Visitor there in the spring of 1956, is an attempt to make the subject accessible to everyone. Primarily it is a textbook for a course at the junior-senior level, but we believe that it can be used with profit also by graduate students. Because the algebra required is not the familiar commutative algebra, a disproportionate amount of the book is given over to necessary algebraic preliminaries. However, this is all to the good because the study of noncommutativity is not only essential for the development of knot theory but is itself an important and not overcultivated field. Perhaps the most fascinating aspect of knot theory is the interplay between geometry and this noncommutative algebra.

For the past thirty years Kurt Reidemeister's Ergebnisse publication *Knotentheorie* has been virtually the only book on the subject. During that time many important advances have been made, and moreover the combinatorial point of view that dominates *Knotentheorie* has generally given way to a strictly topological approach. Accordingly, we have emphasized the topological invariance of the theory throughout.

There is no doubt whatever in our minds but that the subject centers around the concepts: *knot group*, *Alexander matrix*, *covering space*, and our presentation is faithful to this point of view. We regret that, in the interest of keeping the material at as elementary a level as possible, we did not introduce and make systematic use of covering space theory. However, had we done so, this book would have become much longer, more difficult, and

[1] H.L. Frisch and E. Wasserman, "Chemical Topology," *J. Am. Chem. Soc.*, 83 (1961) 3789–3795

presumably also more expensive. For the mathematician with some maturity, for example one who has finished studying this book, a survey of this central core of the subject may be found in Fox's "A quick trip through knot theory" (1962).[1]

The bibliography, although not complete, is comprehensive far beyond the needs of an introductory text. This is partly because the field is in dire need of such a bibliography and partly because we expect that our book will be of use to even sophisticated mathematicians well beyond their student days. To make this bibliography as useful as possible, we have included a *guide to the literature*.

Finally, we thank the many mathematicians who had a hand in reading and criticizing the manuscript at the various stages of its development. In particular, we mention Lee Neuwirth, J. van Buskirk, and R. J. Aumann, and two Dartmouth undergraduates, Seth Zimmerman and Peter Rosmarin. We are also grateful to David S. Cochran for his assistance in updating the bibliography for the third printing of this book.

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[1] See Bibliography

Contents

Chapter VI · Presentation of a Knot Group

Chapter VII · The Free Calculus and the Elementary Ideals

Chapter VIII · The Knot Polynomials

Chapter IX · Characteristic Properties of the Knot Polynomials