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Gaussian Random Processes

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To Andrei Nickolajevich Kolmogorov

Preface

The book deals mainly with three problems involving Gaussian stationary processes. The first problem consists of clarifying the conditions for mutual absolute continuity (equivalence) of probability distributions of a "random process segment" and of finding effective formulas for densities of the equivalent distributions. Our second problem is to describe the classes of spectral measures corresponding in some sense to regular stationary processes (in particular, satisfying the well-known "strong mixing condition") as well as to describe the subclasses associated with "mixing rate". The third problem involves estimation of an unknown mean value of a random process, this random process being stationary except for its mean, i.e., it is the problem of "distinguishing a signal from stationary noise". Furthermore, we give here auxiliary information (on distributions in Hilbert spaces, properties of sample functions, theorems on functions of a complex variable, etc.).

Since 1958 many mathematicians have studied the problem of equivalence of various infinite-dimensional Gaussian distributions (detailed and systematic presentation of the basic results can be found, for instance, in [23]). In this book we have considered Gaussian stationary processes and arrived, we believe, at rather definite solutions.

The second problem mentioned above is closely related with problems involving ergodic theory of Gaussian dynamic systems as well as prediction theory of stationary processes. From a probabilistic point of view, this problem involves the conditions for weak dependence of the "future" of the process on its "past". The employment of these conditions has resulted in a fruitful theory of limit theorems for weakly dependent variables (see, for instance, [14], [22]); the best known condition of this kind is obviously the so-called condition of "strong mixing". The problems arising in considering regularity conditions reduce in the case of Gaussian processes to a peculiar approximation problem related to linear spectral theory. The book contains the results of investigations of this problem which helped solve it almost completely.

The problem of estimating the mean is perhaps the oldest and most widely known in mathematical statistics. There are two approaches to the solution of this problem: first, the best unbiased estimates can be constructed on the basis of the spectral density of stationary noise; otherwise the least squares method can be applied.

We suggest one common class of "pseudobest" estimates to include best unbiased estimates as well as classical least squares estimates. For these "pseudobest" estimates explicit expressions are given, consistency conditions are found, asymptotic formulas are derived for the error correlation matrix, and conditions for asymptotic effectiveness are defined. It should be mentioned that the results relevant to regularity conditions and the mean estimation are formulated in spectral terms and can automatically be carried over (within the "linear theory") to arbitrary wide-sense stationary processes.

Each chapter has its own numbering of formulas, theorems, etc. For example, formula (4.21) means formula 21 of Section 4 of the same chapter where the reference is made. For the convenience of the reader we provide references to textbooks or reference books. The references are listed at the end of the book.

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