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Bent Jørgensen

Statistical Properties of
the Generalized Inverse
Gaussian Distribution



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Preface

In 1978 the idea of studying the generalized inverse Gaussian distribution was proposed to me by Professor Ole Barndorff-Nielsen, who had come across the distribution in the study of the so-called hyperbolic distributions where it emerged in connection with the representation of the hyperbolic distributions as mixtures of normal distributions. The statistical properties of the generalized inverse Gaussian distribution were at that time virtually undeveloped, but it turned out that the distribution has some nice properties, and models many sets of data satisfactorily. This work contains an account of the statistical properties of the distribution as far as they are developed at present.

The work was done at the Department of Theoretical Statistics, Aarhus University, mostly in 1979, and was partial fulfilment towards my M.Sc. degree. I wish to convey my warm thanks to Ole Barndorff-Nielsen and Preben Blæsild for their advice and for comments on earlier versions of the manuscript and to Jette Hamborg for her skilful typing.

Bent Jørgensen

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