PART II

EXTREMAL PROPERTIES OF DEPENDENT SEQUENCES

In Chapters 3–6, which comprise Part II, we focus on the effects of dependence among $\xi_1, \ldots, \xi_n$, on the classical extremal results. For the most part it will be assumed that the ξ's, while dependent, are identically distributed, and in fact form a strictly stationary sequence. However, some important non-stationary cases will also be briefly considered.

The task of Chapter 3 is to generalize basic results concerning the maximum M_n, to apply to stationary sequences. As will be seen, the generalization follows in a rather complete way under certain natural restrictions limiting the dependence structure of the sequence. In particular, it is shown that under these restrictions the limit laws in such dependent cases are precisely the same as the classical ones, and indeed, in a given dependent case, that the same limiting law applies as would if the sequence were *independent*, with the same marginal distribution. Some results and examples of non-normal sequences where this is not true are also given.

In Chapter 4 this theory is applied to the case of stationary *normal* sequences. It is found there that the dependence conditions required are satisfied under very weak restrictions on the covariances associated with the sequence.

Chapter 5 is concerned with the topic of Chapter 2—namely, the properties of $M_n^{(k)}$, the kth largest of the ξ_i. The discussion is approached through a consideration of the "point process of exceedances of a level u_n" by the sequence $\xi_1, \xi_2, \ldots$. This provides what we consider to be a helpful and illuminating viewpoint. In particular, a simple convergence theorem shows the Poisson nature of the exceedances of high levels, leading to the desired generalizations of the classical results for $M_n^{(k)}$.

Two topics complementing the theory for normal sequences are dealt with in Chapter 6. In the first the previous extremal results are shown

(appropriately modified) to apply to a class of nonstationary normal sequences. This, in particular, provides the asymptotic distribution of the maximum of a stationary normal sequence to which an appropriate trend, or seasonal component, has been added. The second topic concerns stationary normal sequences under very strong dependence. There is no complete theory for this case but a variety of different limiting results are exhibited.