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EXTREMES
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Preface

Classical Extreme Value Theory—the asymptotic distributional theory for maxima of independent, identically distributed random variables—may be regarded as roughly half a century old, even though its roots reach further back into mathematical antiquity. During this period of time it has found significant application—exemplified best perhaps by the book *Statistics of Extremes* by E. J. Gumbel—as well as a rather complete theoretical development.

More recently, beginning with the work of G. S. Watson, S. M. Berman, R. M. Loynes, and H. Cramér, there has been a developing interest in the extension of the theory to include, first, dependent sequences and then continuous parameter stationary processes. The early activity proceeded in two directions—the extension of general theory to certain dependent sequences (e.g., Watson and Loynes), and the beginning of a detailed theory for stationary sequences (Berman) and continuous parameter processes (Cramér) in the normal case.

In recent years both lines of development have been actively pursued. It has proved possible to unify the two directions and to give a rather complete and satisfying general theory along the classical lines, including the known results for stationary normal sequences and processes as special cases. A principal aim of this work is to present this theory in as complete and up-to-date a form as possible, alongside a reasonably comprehensive discussion of the classical case. The treatment is thus unified with regard to both the classical and dependent cases, and also in respect to consideration of normal and more general stationary sequences and processes.

Closely related to the properties of extremes are those of exceedances and upcrossings of high levels, by sequences and continuous parameter processes. By regarding such exceedances and upcrossings as point processes, one may obtain some quite general results demonstrating convergence to Poisson and related point processes. A number of interesting results follow concerning the asymptotic behaviour of the magnitude and location of such quantities as the kth largest maxima (or local maxima, in the continous setting). These and a number of other related topics have been taken up, especially for continuous parameter cases.

The volume is organized in four parts. Part I provides a reasonably comprehensive account of the central distributional results of classical extreme value theory—surrounding the Extremal Types Theorem. We have attempted to make this quite straightforward, using relatively elementary methods, and to highlight the main ideas on which the later extensions to dependent cases are based.

Part II contains the basic extension of the classical theory applying to stationary sequences and to some important nonstationary cases. The main key to this work is the appropriate restriction of dependence between widely separated members of the sequence, so that the classical limits still hold. Normal sequences are particularly emphasized and provide illuminating examples of the roles played by the various assumptions.

In Part III we turn to continuous parameter cases. The emphasis in this part is on stationary normal processes, which, for clarity, we treat in some detail before giving the general theory surrounding the Extremal Types Theorem. In addition to extremal theory, this part concerns properties of local maxima, point processes of upcrossings, models for local behaviour, and related topics.

Finally, Part IV contains specific applications of (and small extensions to) the theory for particular, real situations. Since the theory largely predicts the same extremal behaviour as in the classical case, there is limited usefulness in providing data which simply illustrate this well. Rather, we have tried to grapple with typical practical issues and problems which arise in putting theory to work. We have not attempted systematic case studies, but have primarily selected examples which involve interesting facets, and raise issues that demand thoughtful consideration.

Many of the results given here have appeared in print in various forms, but a number are hitherto unpublished. Most of the contents of this work may be easily understood by a reader who has taken a (non-measure-theoretic) introductory graduate probability course. Possible exceptions include the material on point process convergence (the details being given in an appendix), but even for this we feel that a reader should be able to obtain a good intuitive understanding from the text.

It is indeed a pleasure to acknowledge the support of the U.S. Office of Naval Research, the Swedish Natural Science Research Council, the Danish Natural Science Research Council, and the Swedish Institute of Applied Mathematics, in much of the research leading to this work. We are also most grateful to Drs. Jacques de Maré and Jan Lanke for their help and suggestions on various aspects of this project and to Dr. Olav Kallenberg for his illuminating comments. We thank a number of readers including J. Castellana, B. Collings, R. Frimmel, N. Gerr, P. Hougaard, D. Kikuchi, I. McKeague, E. Murphree, and I. Skovgaard for their suggestions for improvement in clarity of the text; Ruth Bahr, Anitha Bergdahl, Betty Blake, Dagmar Jensen, Ingalill Karlsson, Anna Morén, Beatrice Tuma, and Ingrid Westerberg for their splendid typing of this volume and its earlier manuscript versions, and Sten Lindgren for preparing several of the figures.

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