PART I

CLASSICAL THEORY OF EXTREMES

Classical extreme value theory is concerned substantially with distributional properties of the maximum

$$M_n = \max(\xi_1, \xi_2, \ldots, \xi_n)$$

of n independent and identically distributed random variables, as n becomes large. In Part I we have attempted to give a relatively comprehensive account of the central distributional results of the classical theory, using the simplest available proofs, and emphasizing their general features which lead to subsequent extensions to dependent situations.

Two results of basic importance are proved in Chapter 1. The first is the fundamental result—here called the Extremal Types Theorem—which exhibits the possible limiting forms for the distribution of M_n under linear normalizations. More specifically, this basic classical result states that if for some sequences of normalizing constants $a_n > 0$, b_n, $a_n(M_n - b_n)$ has a nondegenerate limiting distribution function $G(x)$, then G must have one of just three possible "forms". The three "extreme value distributions" involved were discovered by Fréchet, and Fisher and Tippett, and discussed more completely later by Gnedenko. Here we use more recent proofs, substantially simplified by the use of techniques of de Haan.

The second basic result given in Chapter 1 is almost trivial in the independent context, and gives a simple necessary and sufficient condition under which $P\{M_n \le u_n\}$ converges, for a given sequence of constants $\{u_n\}$. This result plays an important role here and also in dependent cases, where it is by no means as trivial but still holds under appropriate conditions. Its importance will be seen in Chapter 1 in the development of the classical theory given there for the domains of attraction to the three extreme value types. The theory is illustrated by several examples from each of the possible limiting types and the chapter is concluded with a brief corresponding discussion of minima.

The theme of Chapter 2 is the corresponding limiting distributions for the kth largest $M_n^{(k)}$ of $\xi_1, \ldots, \xi_n$, where k may be fixed or tend to infinity with n. The case for fixed k (when $M_n^{(k)}$ is an "extreme order statistic") is of primary concern and is discussed by means of asymptotic Poisson properties of the exceedances of high levels by the sequence $\xi_1, \xi_2, \ldots$. These properties, which here involve simply the convergence of binomial to Poisson distributions will recur in more interesting and sophisticated forms in the later parts of the volume. Rather efficient and transparent estimates for the rate of convergence in the limit theorems are also presented in this chapter.

Finally, some description is given of the available theory for cases when $k = k_n$ tends to infinity with n (involving "central" and "intermediate" order statistics). This discussion is included for completeness only and will not be developed in the subsequent dependent context.