

# Open Problems in Communication and Computation

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Editors

# Open Problems in Communication and Computation

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*Cover illustration:* A Julia set mapped onto the unit sphere from the complex plane; generated by Christian Götze in IC\* Laboratory at Bellcore.

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