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Preface to the Second Edition

I have taken the opportunity afforded by the appearance of a second edition of *Geophysical Fluid Dynamics* to make a number of revisions and additions to the text. Since the purpose of the book is largely pedagogic, the changes are intended to strengthen the treatment of the fundamentals of the subject rather than bringing up to the current moment the results of ongoing research in the field as would be appropriate for a research monograph. Chapters 3 and 4 now contain a discussion of the elements of geostrophic turbulence whose treatment was lacking in the first edition. In Chapter 5, I have substantially replaced Section 5.13 with a new discussion of the role of topography in the wind-driven circulation with an example of its influence much more striking than earlier. Chapter 6 is the most extensively revised section of the book. Sections on wave-mean flow interaction and the theory of the thermocline have been completely rewritten reflecting the increase in understanding of these fundamental subjects since the preparation of the first edition. Chapter 6 also contains a new derivation of the planetary and synoptic scale geostrophic potential vorticity equations. The new derivation supplements the more traditional derivation of the first edition and illuminates the relationship between the two regimes. In Chapter 7, I have reworked the section on the classical Charney problem in a way which should considerably simplify and illuminate the essential aspects of the problem. The discussion of the finite amplitude problem has been completely reworked to offer the reader an introduction to the role of dissipation in the nonlinear dynamics and the appearance of limit cycle and chaotic behavior. Finally, Chapter 7

has a new section describing some useful theorems on the instability of nonparallel flows.

I am indebted to several correspondents for pointing out errors in the first edition which are corrected herein. The preparation of this second edition was done during a sabbatical year in Venice, Italy.

I gratefully acknowledge the continuing support of the Woods Hole Oceanographic Institution during the period of the sabbatical. I wish also to express my gratitude to the staff of the Istituto per lo Studio della Dinamica delle Grandi Masse for their warm hospitality during the period of the preparation of the revisions for this book. To paraphrase Henry James on the subject, to be able to work and live in Venice led to perhaps the greatest state of happiness consistent with the preservation of reason.

Woods Hole
September 1986

Joseph Pedlosky

Preface to the First Edition

The content of this book is based, largely, on the core curriculum in geophysical fluid dynamics which I and my colleagues in the Department of Geophysical Sciences at The University of Chicago have taught for the past decade. Our purpose in developing a core curriculum was to provide to advanced undergraduates and entering graduate students a coherent and systematic introduction to the theory of geophysical fluid dynamics. The curriculum and the outline of this book were devised to form a sequence of courses of roughly one and a half academic years (five academic quarters) in length. The goal of the sequence is to help the student rapidly advance to the point where independent study and research are practical expectations. It quickly became apparent that several topics (e.g., some aspects of potential theory) usually thought of as forming the foundations of a fluid-dynamics curriculum were merely classical rather than essential and could be, however sadly, dispensed with for our purposes. At the same time, the diversity of interests of our students is so great that no curriculum can truly be exhaustive in such a curriculum period. It seems to me that the best that can be achieved as a compromise is a systematic introduction to some important segment of the total scope of geophysical fluid dynamics which is illustrative of its most fruitful methods. The focus of this book is thus the application of fluid mechanics to the dynamics of large-scale flows in the oceans and the atmosphere. The overall viewpoint taken is a theoretical, unified approach to the study of both the atmosphere and the oceans.

One of the key features of geophysical fluid dynamics is the need to combine approximate forms of the basic fluid-dynamical equations of

motion with careful and precise analysis. The approximations are required to make any progress possible, while precision is demanded to make the progress meaningful. This combination is often the most elusive feature for the beginning student to appreciate. Therefore, much of the discussion of this book is directed towards the development of the basic notions of scaling and the subsequent derivation of systematic approximations to the equations of motion. The union of physical and intuitive reasoning with mathematical analysis forms the central theme. The ideas of geostrophic scaling, for example, are repeated several times, in various contexts, to illustrate the ideas by example.

The development of physical intuition is always a slow process for the beginner, and the book has a structure which aims to ease that important process. Chapters 1 and 2 discuss certain elementary but fundamental ideas in general terms before the complexities of scaling are required. In Chapter 3 the inviscid dynamics of a homogeneous fluid is discussed in order to expose, in the simplest context, the nature of quasigeostrophic motion. It has been my experience that the absence of the complexities necessarily associated with density stratification is a great help in penetrating quickly to rather basic concepts of potential vorticity dynamics. Rossby waves, inertial boundary currents, the β -plane, energy propagation, and wave interaction, etc. are all topics whose first treatment is clearer and simpler for the fluid of constant density. Similarly, Chapter 4 describes some of the simple ideas of the influence of friction on large-scale flows in the context of a homogeneous fluid. The vexing problem of turbulence receives short shrift here. Only the simplest model of turbulent mixing is formulated. It is my view that, unsatisfactory as such a model is as a theory of turbulence, it is sufficient for the purposes to which it is generally applied in the theory of large-scale flows. Chapter 5 serves to exemplify the use of the homogeneous model in the discussion of a problem of major geophysical interest, i.e., the wind-driven ocean circulation.

Chapter 6 has two main purposes. First is the systematic development of the quasigeostrophic dynamics of a stratified fluid for flow on a sphere. Careful attention is given to the development of the β -plane model on logical and straightforward lines. I believe many of the elements of the derivation have been hitherto unfortunately obscure. The second major goal is the application of quasigeostrophic dynamics to a few problems which I feel are central to both meteorology and oceanography and whose outlines, at least, should be familiar to the serious student.

Chapter 7 is reserved for instability theory. Since the publication of the pioneering papers of Charney and Eady, instability theory has held a central position in the conceptual foundation of dynamic meteorology. Recent advances in oceanography suggest a significant role for instability theory also in oceanic dynamics. Baroclinic and barotropic instability are both discussed in Chapter 7, not exhaustively, but to the degree I feel is necessary to provide a clear picture of the basic issues. The final chapter discusses certain topics, not easily grouped into the broad categories of earlier chap-

ters, and chosen primarily to illustrate the way in which the ideas previously developed can be extended by similar methods.

The task of writing a text is made especially difficult by the evident impossibility of being truly comprehensive. The limitations of size make it necessary to omit topics of interest. To begin with, certain introductory aspects of fluid mechanics, such as the derivation of the Navier–Stokes equations (which is essential to a core curriculum) are deleted. Such topics may be found already in such excellent texts as Batchelor’s *Fluid Dynamics* or Sommerfeld’s *Mechanics of Deformable Bodies*. In other cases, when confronted by difficult choices, I have tried to include material which illustrates principles of general utility in fluid mechanics, e.g., boundary-layer concepts and the application of multiple-time-scale ideas to nonlinear problems. In this way I believe that the problems of geophysical fluid dynamics serve additionally as an excellent vehicle for the teaching of broader dynamical concepts. For example, the relationship between group velocity and phase speed in the Rossby wave is discussed at length in Chapter 3. There is, perhaps, no more dramatic example of the distinction between the two concepts in all fluid dynamics, and it can serve as a useful example of such a distinction for students of varying fluid-dynamical interests.

Naturally, in many cases I have chosen topics for discussion on the basis of my own interest and judgement. To that extent the text is a personal expression of my view of the subject.

It was my happy good fortune as a student to have had a series of marvelous teachers of fluid dynamics. Each in their own way made the subject vivid and beautiful to me. By now, no doubt, many of their ideas and attitudes are so intimately mixed into my own view that they appear implicitly here to the benefit of the text.

It is a pleasure, however, to explicitly acknowledge the singular influence of my teacher and colleague, Professor Jule Charney. His prodigious contributions to the study of the dynamics of the atmosphere and oceans as well as his example of scholarly integrity have been a continuing source of inspiration.

This book was largely written during a sabbatical year made possible by a fellowship from the John Simon Guggenheim Foundation, as well as by the continued support of the University of Chicago. The Woods Hole Oceanographic Institution kindly provided an office for me for the year and their warm hospitality considerably eased the task of writing and preparing the original manuscript. Special thanks are due to the students of the M.I.T.–Woods Hole joint program in physical oceanography, who read the evolving manuscript and made numerous helpful corrections and suggestions. Doris Haight typed the manuscript with skill, patience, and good humor.

Woods Hole
September 1979

Joseph Pedlosky

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