Applied Mathematical Sciences
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Applied Mathematical Sciences

1. *John:* Partial Differential Equations, 4th ed.
2. *Sirovich:* Techniques of Asymptotic Analysis.
3. *Hale:* Theory of Functional Differential Equations, 2nd ed.
4. *Percus:* Combinatorial Methods.
5. *von Mises/Friedrichs:* Fluid Dynamics.
6. *Freiberger/Grenander:* A Short Course in Computational Probability and Statistics.
7. *Pipkin:* Lectures on Viscoelasticity Theory.
8. *Giacoglia:* Perturbation Methods in Non-linear Systems.
9. *Friedrichs:* Spectral Theory of Operators in Hilbert Space.
10. *Stroud:* Numerical Quadrature and Solution of Ordinary Differential Equations.
11. *Wolovich:* Linear Multivariable Systems.
12. *Berkovitz:* Optimal Control Theory.
13. *Bluman/Cole:* Similarity Methods for Differential Equations.
14. *Yoshizawa:* Stability Theory and the Existence of Periodic Solution and Almost Periodic Solutions.
15. *Braun:* Differential Equations and Their Applications, 3rd ed.
16. *Lefschetz:* Applications of Algebraic Topology.
17. *Collatz/Wetterling:* Optimization Problems.
18. *Grenander:* Pattern Synthesis: Lectures in Pattern Theory, Vol. I.
19. *Marsden/McCracken:* Hopf Bifurcation and Its Applications.
20. *Driver:* Ordinary and Delay Differential Equations.
21. *Courant/Friedrichs:* Supersonic Flow and Shock Waves.
22. *Rouche/Habets/Laloy:* Stability Theory by Liapunov's Direct Method.
23. *Lamperti:* Stochastic Processes: A Survey of the Mathematical Theory.
24. *Grenander:* Pattern Analysis: Lectures in Pattern Theory, Vol. II.
25. *Davies:* Integral Transforms and Their Applications, 2nd ed.
26. *Kushner/Clark:* Stochastic Approximation Methods for Constrained and Unconstrained Systems.
27. *de Boor:* A Practical Guide to Splines.
28. *Keilson:* Markov Chain Models—Rarity and Exponentiality.
29. *de Veubeke:* A Course in Elasticity.
30. *Shiatycki:* Geometric Quantization and Quantum Mechanics.
31. *Reid:* Sturmian Theory for Ordinary Differential Equations.
32. *Meis/Markowitz:* Numerical Solution of Partial Differential Equations.
33. *Grenander:* Regular Structures: Lectures in Pattern Theory, Vol. III.
34. *Kevorkian/Cole:* Perturbation Methods in Applied Mathematics.
35. *Carr:* Applications of Centre Manifold Theory.
36. *Bengtsson/Ghil/Källén:* Dynamic Meteorology: Data Assimilation Methods.
37. *Saperstone:* Semidynamical Systems in Infinite Dimensional Spaces.
38. *Lichtenberg/Lieberman:* Regular and Chaotic Dynamics, 2nd ed.
39. *Piccini/Stampacchia/Vidossich:* Ordinary Differential Equations in $\mathbf{R}^n$.
40. *Naylor/Sell:* Linear Operator Theory in Engineering and Science.
41. *Sparrow:* The Lorenz Equations: Bifurcations, Chaos, and Strange Attractors.
42. *Guckenheimer/Holmes:* Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields.
43. *Ockendon/Taylor:* Inviscid Fluid Flows.
44. *Pazy:* Semigroups of Linear Operators and Applications to Partial Differential Equations.
45. *Glashoff/Gustafson:* Linear Operations and Approximation: An Introduction to the Theoretical Analysis and Numerical Treatment of Semi-Infinite Programs.
46. *Wilcox:* Scattering Theory for Diffraction Gratings.
47. *Hale et al:* An Introduction to Infinite Dimensional Dynamical Systems—Geometric Theory.
48. *Murray:* Asymptotic Analysis.
49. *Ladyzhenskaya:* The Boundary—Value Problems of Mathematical Physics.
50. *Wilcox:* Sound Propagation in Stratified Fluids.
51. *Golubitsky/Schaeffer:* Bifurcation and Groups in Bifurcation Theory, Vol. I.

(continued following index)

Carlo Marchioro Mario Pulvirenti

Mathematical Theory of Incompressible Nonviscous Fluids

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Preface

Fluid dynamics is an ancient science incredibly alive today. Modern technology and new needs require a deeper knowledge of the behavior of real fluids, and new discoveries or steps forward pose, quite often, challenging and difficult new mathematical problems. In this framework, a special role is played by incompressible nonviscous (sometimes called *perfect*) flows. This is a mathematical model consisting essentially of an evolution equation (the Euler equation) for the velocity field of fluids. Such an equation, which is nothing other than the Newton laws plus some additional structural hypotheses, was discovered by Euler in 1755, and although it is more than two centuries old, many fundamental questions concerning its solutions are still open. In particular, it is not known whether the solutions, for reasonably general initial conditions, develop singularities in a finite time, and very little is known about the long-term behavior of smooth solutions. These and other basic problems are still open, and this is one of the reasons why the mathematical theory of perfect flows is far from being completed.

Incompressible flows have been attached, by many distinguished mathematicians, with a large variety of mathematical techniques so that, today, this field constitutes a very rich and stimulating part of applied mathematics. The idea of writing the present book was motivated by the fact that, although there are many interesting books on the subject, no recent one, to our knowledge, is oriented toward mathematical physics. By this we mean a book that is mathematically rigorous and as complete as possible without hiding the underlying physical ideas, presenting the arguments in a natural order, from basic questions to more sophisticated ones, proving everything and trying, at the same time, to avoid boring technicalities. This is our purpose.

The book does not require a deep mathematical knowledge. The required background is a good understanding of the classical arguments of mathematical analysis, including the basic elements of ordinary and partial differential equations, measure theory and analytic functions, and a few notions of potential theory and functional analysis.

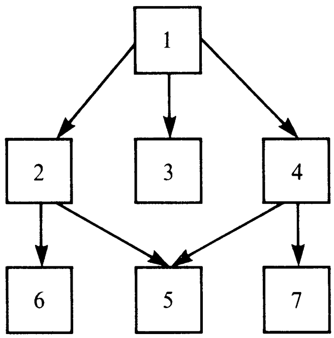
The exposition is as self-contained as possible. Several appendices, devoted to technical or elementary classical arguments, are included. This does not mean, however, that the book is easy to read. In fact, even if we tried to present the topics in an elementary fashion and in the simplest cases, the style is, in general, purely mathematical and rather concise, so that the reader quite often is requested to spend some time in independent thinking during the most delicate steps of the exposition. Some exercises, with a varying degree of difficulty (the most difficult are marked by *), are presented at the end of many chapters. We believe solving them is the best test to see whether the basic notions have been understood.

The choice of arguments is classical and in a sense obligatory. The presentation of the material, the relative weight of the various arguments, and the general style reflect the tastes of the authors and their knowledge. It cannot be otherwise.

The material is organized as follows: In Chapter 1 we present the basic equations of motion of incompressible nonviscous fluids (the Euler equation) and their elementary properties. In Chapter 2 we discuss the construction of the solutions of the Cauchy problem for the Euler equation. In Chapter 3 we study the stability properties of stationary solutions. In Chapter 4 we introduce and discuss the vortex model. In Chapter 5 we briefly analyze the approximation schemes for the solutions of fluid dynamical equations. Chapter 6 is devoted to the time evolution of discontinuities such as the vortex sheets or the water waves. Finally, in Chapter 7 we discuss turbulent motions. This last chapter mostly contains arguments of current research and is essentially discursive.

The final section of each chapter is generally devoted to a discussion of the existing literature and further developments. We hope that this will stimulate the reader to study and research further.

The book can be read following the natural order of the chapters, but also along the following paths:



A possible criticism of the book is that two-dimensional flows are treated in much more detail than three-dimensional ones, which are, physically speaking, much more interesting. Unfortunately, for a mathematical treatise, it cannot be otherwise: The mathematical theory of a genuine three-dimensional flow is, at present, still poor compared with the rather rich analysis of the two-dimensional case to which we address many efforts.

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Contents