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M. Bala Subrahmanyam

*Finite Horizon H_∞
and Related Control Problems*

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Preface

THIS book presents a generalized state-space theory for the analysis and synthesis of finite horizon suboptimal H_∞ controllers. We derive expressions for a suboptimal controller in a general setting and propose an approximate solution to the H_∞ performance robustness problem. The material in the book is taken from a collection of research papers written by the author.

The book is organized as follows. Chapter 1 treats nonlinear optimal control problems in which the cost functional is of the form of a quotient or a product of powers of definite integrals. The problems considered in Chapter 1 are very general, and the results are useful for the computation of the actual performance of an H_∞ suboptimal controller. Such an application is given in Chapters 4 and 5. Chapter 2 gives a criterion for the evaluation of the infimal H_∞ norm in the finite horizon case. Also, a differential equation is derived for the achievable performance as the final time is varied. A general suboptimal control problem is then posed, and an expression for a suboptimal H_∞ state feedback controller is derived. Chapter 3 develops expressions for a suboptimal H_∞ output feedback controller in a very general case via the solution of two dynamic Riccati equations. Assuming the adequacy of linear expressions, Chapter 4 gives an iterative procedure for the synthesis of a suboptimal H_∞ controller that yields the required performance even under parameter variations. As a by-product, an expression for the variation of performance due to parameter variations is given for this specific controller. Chapter 5 treats a general minimization problem in which the cost functional is a quotient of definite integrals. The results are useful in computing the actual performance of a given controller. Also, an expression is given for the variation of performance in terms of variations in the system matrices. In

Chapter 6, a design of the F/A-18A Automatic Carrier Landing System is accomplished using finite horizon H_∞ techniques. Only longitudinal equations of motion are considered, and a suboptimal output feedback controller is synthesized. The object of the design is to maintain a constant flight path angle under worst-case conditions of vertical gust and sensor noise.

I take this opportunity to express my gratitude to the management of the Naval Air Warfare Center for the support of this research. Thanks are also due to my wife Carol and our two year old daughter Susan Rebekah for their love and emotional support. Finally, I would like to give special recognition to our newborn son Matthew Aaron whose arrival fortuitously coincided with the completion of this monograph.

Warminster, Pennsylvania
January 27, 1995

— M. B. S.