IV

Relations to Logical and Computer-Algebraic Calculi

The two contributions in this chapter are situated, so to speak, at opposite ends of the spectrum of calculi. Type theoretic calculi arose originally in the foundations of mathematics, in the context of excluding the set-theoretic paradoxa. They arose again in the context of proof theory and, more recently, as a tool in software checking, from which grew an active branch of theoretical computer science concerned mainly with polymorphic types. Computer-algebraic calculi obviously have their roots in algebra as a discipline of formal operations on mathematical objects. One of the most successful branches deals with calculi for formal integration and closed-form solutions of differential equations, starting with the famous theorems of Liouville, Ritt, Risch, and many others. Again, this has become a very active branch of computer science.

Both of these calculi are addressed here from a point of view closely connected to the "combinatory programme": Daniel Otth shows in his contribution that it is possible to interpret various current type calculi in a very transparent way as combinatory calculi, thereby simplifying and extending the subject in a new direction. Martin von Mohrenschildt also extends the discipline of formal integration by admitting function symbols that denote discontinuous functions, which, in fact, arose originally from combinatory algebra considerations.