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Preface

Combinatory logic started as a programme in the foundation of mathematics and in an historical context at a time when such endeavours attracted the most gifted among the mathematicians. This small volume arose under quite different circumstances, namely within the context of reworking the mathematical foundations of computer science.

I have been very lucky in finding gifted students who agreed to work with me and chose, for their Ph.D. theses, subjects that arose from my own attempts to create a coherent mathematical view of these foundations.¹ The result of this collaborative work is presented here in the hope that it does justice to the individual contributor and that the reader has a chance of judging the work as a whole.

E. Engeler
ETH Zurich, April 1994

¹Collected in Chapter III, An Algebraization of Algorithmics, in *Algorithmic Properties of Structures, Selected Papers of Erwin Engeler*, World Scientific Publ. Co., Singapore, 1993, pp. 183–257.