

Editorial Policy

for the publication of monographs

In what follows all references to monographs are applicable also to multiauthorship volumes such as seminar notes.

§ 1. Lecture Notes aim to report new developments - quickly, informally, and at a high level. Monograph manuscripts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. Furthermore, the manuscripts should provide sufficient motivation, examples, and applications. This clearly distinguishes Lecture Notes manuscripts from journal articles which normally are very concise. Articles intended for a journal but too long to be accepted by most journals usually do not have this "lecture notes" character. For similar reasons it is unusual for Ph.D. theses to be accepted for the Lecture Notes series.

§ 2. Manuscripts or plans for Lecture Notes volumes should be submitted (preferably in duplicate) either to one of the series editors or to Springer-Verlag, New York. These proposals are then refereed. A final decision concerning publication can only be made on the basis of the complete manuscript, but a preliminary decision can often be based on partial information: a fairly detailed outline describing the planned contents of each chapter, and an indication of the estimated length, a bibliography, and one or two sample chapters - or a first draft of the manuscript. The editors will try to make the preliminary decision as definite as they can on the basis of the available information.

§ 3. Final manuscripts should be in English. They should contain at least 100 pages of scientific text and should include

- a table of contents;
- an informative introduction, perhaps with some historical remarks: it should be accessible to a reader not particularly familiar with the topic treated;
- a subject index: as a rule this is genuinely helpful for the reader.

Lecture Notes in Statistics

96

Edited by S. Fienberg, J. Gani, K. Krickeberg,
I. Olkin, and N. Wermuth



Ibrahim Rahimov

Random Sums and Branching Stochastic Processes

Springer-Verlag
New York Berlin Heidelberg London Paris
Tokyo Hong Kong Barcelona Budapest

Ibrahim Rahimov
The Institute of Mathematics
The Academy of Sciences of the Republic of Uzbekistan
Hodjaev Street, 29,
Tashkent, 700142, Uzbekistan

Department of Statistics
Middle East Technical University
06531, Ankara, Turkey

Library of Congress Cataloging-in-Publication Data Available
Printed on acid-free paper.

© 1995 Springer-Verlag New York, Inc.

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden. The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Camera ready copy provided by the editor.

9 8 7 6 5 4 3 2 1

ISBN-13: 978-0-387-94446-3 e-ISBN-13: 978-1-4612-4216-1
DOI: 10.1007/978-1-4612-4216-1

TO THE MEMORY OF MY

FATHER AND MY MOTHER

Uzbekistan is one of the Central Asiatic Republics of the former Soviet Union. The ancient Uzbek cities Samarkand, Bukhara, Khiva and Tashkent have long been major attractions. They have over 4000 architectural monuments, many of them under UNESCO protection. Uzbekistan is the mother country of M. Khorezmi (Al-Goritm), Abu Raikhon Beruni and Mirzo Ulugbeg, famous mathematicians and astronomers of the Middle Ages. Ibn Sina (Avitsenna) the great medical scientist, philosopher and poet was born here.

CONTENTS

INTRODUCTION	1
CHAPTER I. SUMS OF A RANDOM NUMBER OF RANDOM VARIABLES.	5
§1.1. Sampling sums of dependent variables and mixtures of infinitely divisible distributions.	5
§1a. Sums of a random number of random variables.	7
§1b. Multiple sums of dependent random variables.	9
§1c. Sampling sums from a finite population.	14
§1.2. Limit theorems for a sum of randomly indexed sequences.	18
§2a. Sufficient conditions.	18
§2b. Necessary and sufficient conditions.	21
§2c. An application.	25
§1.3. Necessary and sufficient conditions and limit theorems for sampling sums.	27
§3a. Convergence theorems.	27
§3b. The rate of convergence.	35
CHAPTER II. BRANCHING PROCESSES WITH GENERALIZED IMMIGRATION.	44
§2.1. Classical models of branching processes.	44
§1a. Bellman-Harris processes.	45
§1b. Moments and extinction probabilities.	46
§1c. Asymptotics of non-extinction probability and exponential limit distribution.	48
§1d. Branching processes with stationary immigration.	51
§1e. Continuous time branching processes with immigration.	54
§2.2. General branching processes with reproduction dependent immigration.	58
§2a. The model.	59
§2b. The main theorem.	62
§2c. The proof of the main theorem.	64
§2d. Applications of the main theorem.	71
§2.3. Discrete time processes.	76

§3a. The model.	76
§3b. Limit theorems for discrete time processes.	78
§3c. Some examples.	84
§3d. Randomly stopped immigration.	87
§2.4. Convergence to Jirina processes and transfer theorems for branching processes.	92
§4a. The model.	92
§4b. The main theorem and corollaries.	94
§4c. The proof of the main theorem.	97
CHAPTER III. BRANCHING PROCESSES WITH TIME-DEPENDENT IMMIGRATION.	105
§3.1. Decreasing immigration.	105
§1a. The main theorem.	106
§1b. The proof of the main theorem.	118
§1c. State-dependent immigration.	122
§3.2. Increasing immigration.	124
§2a. The process with infinite variance.	124
§2b. The process with finite variance.	132
§3.3. Local limit theorems.	136
§3a. Occupation of an increasing state.	136
§3b. Occupation of a fixed state.	154
CHAPTER IV. THE ASYMPTOTIC BEHAVIOR OF FAMILIES OF PARTICLES IN BRANCHING PROCESSES.	156
§4.1. Sums of dependent indicators.	157
§1a. Sums of functions of independent random variables.	157
§1b. Sampling sums of dependent indicators.	163
§4.2. Family of particles in critical processes.	167
§2a. The model.	167
§2b. Limit theorems.	168
§4.3. Families of particles in supercritical and subcritical processes.	177
§3a. Supercritical processes.	177
§3b. Subcritical processes.	184
REFERENCES.	184
INDEX.	194