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Bernard Aupetit

A Primer on Spectral Theory

Springer-Verlag
New York Berlin Heidelberg London
Paris Tokyo Hong Kong Barcelona

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Mathematics Subject Classifications: Primary—46H, 46J, 46K, 47A; Secondary—31A, 32F

Printed on acid-free paper.

Camera-ready copy supplied by the author using TEX.

9 8 7 6 5 4 3 2 1

ISBN-13:978-0-387-97390-6 e-ISBN-13:978-1-4612-3048-9
DOI: 10.1007/978-1-4612-3048-9

Ce livre est dédié à la mémoire de mes parents.

Marcel Aupetit
Marie-Thérèse Le Charme

"Le cose passate fanno luce alle future, perché il mondo fu sempre di una medesima sorte, e tutto quello che è e sarà è stato in altro tempo; e le cose medesime ritornano ma sotto diversi nomi e colori; però ognuno non le riconosce, ma solo chi è savio e le osserva e considera diligentemente."

Francesco Guicciardini

PREFACE

This book grew out of lectures on spectral theory which the author gave at the Scuola Normale Superiore di Pisa in 1985 and at the Université Laval in 1987. Its aim is to provide a rather quick introduction to the new techniques of subharmonic functions and analytic multifunctions in spectral theory. Of course there are many paths which enter the large forest of spectral theory: we chose to follow those of subharmonicity and several complex variables mainly because they have been discovered only recently and are not yet much frequented. In our book ***Propriétés spectrales des algèbres de Banach***, Berlin, 1979, we made a first incursion, a rather technical one, into these newly discovered areas. Since that time the bushes and the thorns have been cut, so the walk is more agreeable and we can go even further.

In order to understand the evolution of spectral theory from its very beginnings, it is advisable to have a look at the following books: Jean Dieudonné, ***History of Functional Analysis***, Amsterdam, 1981; Antonie Frans Monna, ***Functional Analysis in Historical Perspective***, Utrecht, 1973; and Frédéric Riesz & Béla Szökefalvi-Nagy, ***Leçons d'analyse fonctionnelle***, Budapest, 1952. However the picture has changed since these three excellent books were written. Readers may convince themselves of this by comparing the classical textbooks of Frans Rellich, ***Perturbation Theory***, New York, 1969, and Tosio Kato, ***Perturbation Theory for Linear Operators***, Berlin, 1966, with the present work. They will discover that many of the results found in these books are direct consequences of results, very often more general, given in chapters III, V and VII.

The subharmonicity of the spectrum and the theory of analytic multifunctions also illustrate two striking facts in the history of mathematics. General spectral theory and the abstract theory of subharmonic functions were both invented by F. Riesz (see his *Oeuvres complètes*), but obviously, in the 1920s, he did not realize that the two theories are equivalent in the following sense. Given an analytic family $f(\lambda)$ of operators, the logarithm of the spectral radius of $f(\lambda)$ is subharmonic in

λ, and conversely, given a subharmonic function ϕ, then by Z. Słodkowski's result mentioned on page 167, there exists an analytic family of operators defined on ℓ^2 such that $\phi(\lambda)$ is equal to the logarithm of the spectral radius of $f(\lambda)$. Moreover, in trying to improve some results of F. Hartogs, K. Oka introduced in 1934 the notion of analytic multifunction, but obviously, he saw no link with spectral theory, and the notion remained dormant until the 1980s. So, even in mathematics, the words of the great Florentine statesman and historian Francesco Guicciardini are true.

The book is divided into seven chapters and an appendix. In the first chapter, we give a list of basic results in functional analysis without any proofs, except for those of Machado's theorem and Milman's theorem, which are rarely mentioned in the reference books. The results in this chapter will be used throughout the book.

The second chapter introduces the reader to some examples of bounded operators on Banach spaces and Hilbert spaces, and in particular gives F. Riesz's theory for compact operators and V.I. Lomonosov's theorem on invariant subspaces.

In the third chapter, we define and give examples of Banach algebras. The fundamental Holomorphic Functional Calculus Theorem is then proved. In §4, the subharmonic variation of the spectrum is given with many important applications, for instance the Spectral Maximum Principle (Theorem 3.4.13), Liouville's Spectral Theorem (Theorem 3.4.14), the Holomorphic Variation of Isolated Spectral Values (Theorem 3.4.20), the Subharmonicity of the n-th Diameter (Theorem 3.4.24), the Scarcity of Elements with Finite Spectrum (Theorem 3.4.25), and the Identity Principle for operators having spectra with zero as the only limit point.

The fourth chapter introduces the reader to I.M. Gelfand's theory of commutative Banach algebras and to the representation theory of non-commutative Banach algebras, with the standard results of N. Jacobson and A. Sinclair. At the end we give a new analytic proof of I. Kaplansky's theorem on locally algebraic operators, and we extend it to locally independent operators.

The fifth chapter, along with §4 of chapter III and with chapter VII, is the most important part of the book. It contains a great number of applications of spectral subharmonicity. The most striking applications are the generalization of B.E. Johnson's theorem on the uniqueness of the norm (Theorem 5.5.1), the Perturbation Theorem by Inesessential Elements (Theorem 5.7.4), which immediately implies classical results of I.C. Gohberg, A.F. Ruston and B.A. Barnes, and the new subharmonic proof of B.A. Barnes's theorem on the existence of the socle (Theorem 5.7.8).

The sixth chapter is a classical presentation of the spectral theorem for normal operators on a Hilbert space, with a few applications at the end. In particular it includes the very nice extension of the Russo-Dye theorem obtained by L.T. Gardner. This chapter is independent of chapters V and VII, so it can be read just after chapter IV.

The seventh chapter is the most difficult one for two reasons. Firstly, it involves difficult mathematics, such as the theory of pseudoconvex sets. Secondly, our treatment is superficial. Nevertheless in spite of these drawbacks we have decided to include it in this book in order to induce the reader to learn more on this new subject which has (and certainly will have in the future) very important applications. In particular, the General Pełczyński Conjecture (Theorem 7.2.9) is solved, and applications to the distribution of spectral values in the plane (for instance Theorem 7.3.10) are given.

The appendix is a compendium of all the results needed in the theories of subharmonic functions, of capacity, and of functions of several complex variables. Obviously, no proofs are given because of the strict limit we fixed on the number of pages in our manuscript. They can be found in the standard textboooks mentioned.

Each chapter is followed by a list of problems. In some of them extra material is introduced. A few are very difficult to solve; in these cases we suggest that the reader take a look at the given references.

We have tried to keep the prerequisites to a strict minimum, and to develop the techniques of spectral subharmonicity and analytic multifunctions in the smallest number of pages. This is mainly because we believe that it is easier to learn something quickly and deeply in a small book than to learn in a big book. However the reader is assumed to be familiar with the matter which would generally be covered in one-semester courses on algebra, on complex analysis and on functional analysis. The material presented corresponds roughly to two semesters of lectures, supposing that the students are familiar with the basics of subharmonic functions, capacity and functions of several complex variables. Otherwise it would be better to start with an introductory course covering the matter in the appendix.

Firstly we would like to thank Churchill College, Cambridge, and the Scuola Normale Superiore di Pisa, for their hospitality during the academic year 1984–1985. There we began to write the manuscript of this book. We also thank the Natural Sciences and Engineering Research Council of Canada and the F.C.A.R. Fund of the Province of Quebec for their constant financial help which gave us the opportunity to travel and have interesting discussions with many mathematicians.

Obviously much thanks are due to the many attentive readers of the different versions of the manuscript, in particular to our friend Jaroslav Zemánek, to our former students Line Baribeau and Frédéric Gourdeau, and to our students Daniel Turcotte, Alain Fournier and Abdelaziz Maouche. Special thanks go to Louise Chamberland, Alain Charbonneau and Alain Fournier for the unpleasant work of typing this manuscript.

Quebec
February 1990

Bernard Aupetit

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